Auxiliary Information in Geophysical Inversion

Robert G. Ellis¹, Peter A. Diorio², Ian N. MacLeod¹

Abstract

There are a number of critical auxiliary inputs to a geophysical inversion, apart from the geophysical survey data and explicit geologic constraints, which determine whether the inversion produces a physical property model with real exploration value. Therefore, an understanding nature and effect of the auxiliary inputs is important to anyone involved in geophysical inversion. This presentation focuses on these auxiliary inputs with examples drawn from TMI data collected over the Cannington and San Nicolas deposits. Examples of auxiliary inputs include: assignment of data errors, trend removal, choice of inversion model, weighting functions, amongst others.

¹Geosoft Inc., Toronto, Ontario, Canada
²GeophysicsOne Inc., Oakville, Ontario, Canada

Introduction

Geophysical inverse problems are often tackled by discretizing the earth into a large number of discrete elements and using the governing partial differential equations and regularization to produce a model which simultaneously fits the data and agrees with a priori constraints. A review of the geophysical inversion literature and conference proceedings reveals most publications follow a standard presentation formula: an unconstrained inversion of synthetic data produces approximately the model used to generate the synthetic data, followed by an unconstrained inversion of field data which agrees with a geological model, with the inevitable conclusion that the advertised inversion works. Unfortunately this common and naturally appealing formula is somewhat unsatisfactory. On the most basic level, the geophysical inverse problem is ill-posed primarily by virtue of the well-known fact that different models can give rise the same set of geophysical observations. It is simply not possible to construct a unique model which fits the data without auxiliary information from the end user. Therefore, the success suggested by the standard presentation formula is actually to some degree dependent on the auxiliary information used in the inversion. The same is true for inversion in the real exploration projects with potentially serious consequences. It is the purpose of this work to examine that auxiliary information and the effect it has on the inversion result.

Traditionally, a priori information is considered to be a geological model that the inversion is expected to reproduce as closely as possible subject to fitting the data. In our work we exclude this a priori information from our definition of auxiliary information, because like the observed data itself, it is explicit and obvious to the end user. We take auxiliary information or inputs to include all the remaining factors which influence the outcome of an inversion in a practical exploration context, for example, the assigned data errors, depth weighting functions, regularizations, etc. and even data preprocessing such as trend removal. Indeed, it is the correct application of this auxiliary information that differentiates a good inversion from a poor inversion, and possibly exploration success from exploration failure.

Cannington and San Nicolas

To illustrate the influence of auxiliary information and inputs into a geophysical inversion we use two field datasets: TMI data collected of the Cannington deposit, and TMI data collect over the San Nicolas deposit. The choice of TMI data over base metal deposits is immaterial since our conclusions apply to all geophysical inverse problems.
Briefly, from Walters and Bailey (1998), the Cannington "Broken Hill Type" Ag-Pb-Zn deposit is hosted by the Proterozoic age Eastern Succession of the Mt Isa Inlier, in Queensland, Australia. It occurs under cover and forms a strong, isolated magnetic anomaly, hosted by a sequence of weakly magnetic migmatitic gneisses. Figure 1 shows the TMI response over the deposit, as a grid generated from 200m line spacing airborne data collected at 80m clearance.

Equally briefly, the San Nicolas bimodal-mafic VMS deposit, located in Zacatecas State, Mexico, is hosted in marine volcanic and sedimentary rocks, locally known as the Chilitos Formation, of Upper Jurassic to Lower Cretaceous age, (Johnson et al., 2000). Mineralization is predominately found along a time-line that occurs between the formation of rhyolitic lava domes and the deposition of mafic extrusives above. Figure 2 shows the TMI response over the deposit, as a grid generated from 250m line spacing airborne data, 100m clearance.

The Geophysical Inverse Problem
We begin by assuming that the geophysical inverse problem will be solved using methods based on Tikhonov regularization (Tikhonov 1977). Let represent a geophysical survey performed on physical property model to produce observed data, . One simplified formulation of the geophysical inverse problem can then be written:

where the upper line states that the model is to found by minimizing the total objective function . The total objective function is composed of two terms: the data objective function , and the model objective function, . During the minimization, that the regularization parameter is chosen so that a suitable data misfit is maintained, . The lower line gives the definitions of the data and model
objective functions, and can be chosen in any manner which suits the problem at hand. It is in these choices that the auxiliary inputs come into play.

In defining the data objective function, we have chosen a least squares style misfit measure and explicitly exposed the auxiliary inputs associated with a possible trend, to be removed from the data, and the error, to be assigned to each data point.

In defining the model objective function, we have been deliberately vague. All we have said is that some operator is applied to the difference between the inversion result, and a reference model.

could be chosen to be a simple L2 norm (smallest deviation models), or could involve derivative operators (smooth models), or any other terms thought to be useful (focusing, depth weighting, etc.) By this stage it should be clear that a full study of the auxiliary information going into the inverse problem is far, far beyond the scope of this work. Even the manner in which the single scalar regularization parameter is chosen has been the subject of many books and theses (L-Curve, GCV, misfit, etc.).

In order to make this work as relevant as possible to practicing geophysicists, we restrict our analysis to the effect of a limited, but commonly encountered, subset of auxiliary inputs. Using field data from the Cannington and San Nicolas deposits we focus on those inputs that practicing geophysicists must provide using currently popular inversion software. Our method will be to vary one auxiliary input at a time and observe the effect on the inversion. In the following sections we address auxiliary inputs associated with the data objective function, then move onto choices associated with the model, used in the inversion, and finally with two examples associated with the model objective function operator, appearing in

Data-Based Auxiliary Information
We begin by focusing on the auxiliary information going into the data objective function, and in particular, the choices of data error and trend removal.

Error assignment
Even limiting ourselves to the choice of data error opens up many possibilities, for example, whether the error is a constant value for all data points or depends in some way upon the amplitude of the data values. Here we limit ourselves to the former and examine the effect of the size of the constant error. To illustrate the effect we take the magnetic data collected over the Cannington deposit shown in Figure 1, where the standard deviation of the TMI is 91.6nT. We invert the TMI data with 3 different constant error levels, 20nT, 2nT, and .02nT. The resulting susceptibility models are shown in Figure 3 (left to right respectively). As expected these inversions show that decreasing the error level results in more structure appearing in the model with a corresponding increase in the recovered susceptibility amplitudes. The real point is that the end user must ultimately choose the error level which gives the appropriate trade-off between geologically plausible structure and spurious artefact: our first example of auxiliary input. Before moving on, we note that an enormous body of research exists on automating the process of choosing the correct level of fit to the data, however for geophysical inverse problems, no automatic process can yet replace the assessment of an experienced geoscientist.
Figure 3: The result of inverting the Cannington TMI data using 3 different error levels. On the left \( 20 \text{nT} \) the model is a little too smooth, whereas on the right \( 0.02 \text{nT} \) there are unrealistic artefacts forming in the model due to fitting the noise in the data. Somewhere between these extremes lies the optimum error assignment, perhaps around \( 2 \text{nT} \) (centre).

**Trend removal**

The next auxiliary input we consider is the choice of trend or background response removal from the data, in Eq(1). Geophysical surveys are interested in the variation of a geophysical response and often that variation is superimposed on a background response. A very common geophysical scenario has the response of interest superimposed on the response from a regional structure. In fact, the Cannington response sits in a weak regional response, however, for the purpose of this example, we artificially enhance that regional response to that shown in Figure 4. In Figure 5(left) we show the result of inverting the trend enhanced data with no trend removal, and Figure 5(right) we show the corresponding result with linear trend removal, together with an outline of the known mineralization. Comparing these results shows that the trendremoved data yield a significantly better representation of the target mineralization. What is somewhat surprising is that the inversion with trend left in the data is so little affected by the trend. This illustrates an important phenomenon which will be elaborated and explained later: the padding cells (not shown) absorb the bulk effect of the trend in the data.

Figure 4: Cannington TMI response with enhanced regional trend. The colour scale is the same as that shown in Figure 1. The dotted line indicates the location of the section shown in Figures 3, 5 and 10.
In this section we have examined two data related auxiliary inputs to the inversions, data error and trend removal, in a cursory fashion however it has been sufficient to demonstrate that correct choice of this auxiliary information is critical to producing a good inversion. Next we turn to aspects of the choice of model and model objective function.

**Model Based Auxiliary Input**
By any measure, the main auxiliary input into the geophysical inverse problem enters through the model objective function, Eq (1). However, before turning specifically to the model objective function we emphasize that choice of discretization of model is a form of auxiliary input which can affect the result of the inversion. We begin by considering the effect of model element size, the effect of padding cells, the choice of model type, and finally the geometry of the model elements.

**Model element size**
To examine the effect of model element size on the quality of the inversion result we take the Cannington data shown in Figure 1 and supplement it with a series of target bodies of different size and structure. We then simulate the TMI data from the supplementary target bodies and add it to the Cannington data. This process is illustrated in Figure 6. Next the data shown in Figure 6 were inverted with mesh with cell sizes: 20x20x10m, 40x40x20m, 80x80x40m, and 160x160x80m. The resulting inversion results are shown in Figure 7 clockwise from the top left, respectively. The important point is that the first three inversions yield virtually identical results, including the resolution of the Cannington deposit, with the largest cell size 160m showing a lack of resolution. This has important implications: first, decreasing the cell size will not improve an inversion beyond the structure imposed by the spatial frequencies in the data; and second, an optimum cell size should be predictable from the data. Nevertheless, the choice of cell size is still an auxiliary input to the inversion and must be appropriately chosen.
Figure 6: The Cannington cell size test model: synthetic bodies consisting of pipes, dikes and dipping prisms were added to the Cannington TMI data. Line separation is 200m and flight height is 80m. All bodies have susceptibilities of 0.10 SI except for the dikes which have susceptibility of 0.05 SI. Depth of the bodies varies from 0m to 200m as indicated.

Figure 7: Variable Cell size inversions results shown as isosurfaces. Cell sizes vary from 20x20x10m, 40x40x20m, 80x80x40m, to 160x160x80m, clockwise from top left. There is virtually no difference between the smallest three cell sizes, however resolution is lost with the largest cell size.
Padding Cells

A second model based auxiliary input is associated with the treatment of the response observed in the area of interest but originating from bodies external to the area of interest, often called "edge effects". Edge effects are mitigated by introducing padding cells around the volume of interest. To investigate padding cells as an auxiliary input, we take the Cannington data shown in Figure 1 and add to it the response from two large peripheral "stocks", just outside the area of interest. This process is illustrated in Figure 8.

Figure 8: The "stock" enhanced Cannington TMI data shown as a grid with contours. The two peripheral stocks are shown in red, both have diameter=1km and susceptibility 0.1 SI at depth 100m. The contributions to the TMI from the stocks are evident adjacent to the stocks. Even though the two stocks are identical the effect on their model is different because the inducing field is not vertical (inclination here is 52.6 degrees.)

Figure 9: Inversions of the "stock" enhanced Cannington TMI data shown Figure 8, with 5000m of padding (left) and with 50m of padding (right). When appropriate padding is used very little peripheral "stock" appears in the volume of interest.
Next we invert the stock enhanced data with 5000m of padding, and then with only 50m of padding. The results are shown in Figure 9, which confirm that padding cells are critical for good inversion: when only 50m of padding is used the inversion must incorporate the response from the peripheral stock into the volume of interest, whereas when 5000m of padding is available the effect of the peripheral stock is largely excluded from the volume of interest. The padding cells are extremely effective given the large contribution to the response from the southern stock.

The effectiveness of the padding cells in mitigating edge effects is a hint that they may also play a significant role in mitigating regional trends in the data. Returning to our earlier example, Figure 5, we show in Figure 10 the padding cells corresponding to the models of Figure 5. Observe that leaving the trend in the TMI data has forced the inversion to compensate by adding susceptibility in the padding (Figure 10a), whereas, when the trend is removed from the data before the inversion the padding cells are less critical (Figure 10b), nevertheless padding is still required to supply any higher order regional effects. These examples further emphasize that the choice of padding cells is an important auxiliary inversion input.

**Figure 10: Padding Cells/Trend**

**Element Geometry**

In addition to the cell size, there is also the choice of cell geometry to be made in the defining the inverse problem. This is another important auxiliary input as we shall now demonstrate using the San Nicolas deposit. The deposit occurs in an area with very modest slowly undulating terrain and is covered by volcanic breccia. If the inversion model represents the earth using prism shaped elements, the surface topography must be represented in terms of prisms, giving a step wise approximation to the true topography. A small portion of the topography at San Nicolas thus represented is shown in Figure 11. Unfortunately, the prism topography approximation leads to severe artefacts in any computed geophysical response and corrupts the inversion. Figure 12 shows the RTP response over San Nicolas computed using 10m cells. The ridges due to the prism approximation are clearly evident. By comparison, if the model uses cells which conform to the topography, as illustrated in Figure 13 then a more accurate response will result, leading to a better inversion. This example illustrates that cell geometry is another important auxiliary inversion input.
Element Type

A critically important auxiliary input to a geophysical inverse problem is the choice an appropriate model element type. As an example, consider the inversion of magnetic field data collected over a prospect which has a component of remanent magnetization, magnetic anisotropy, or demagnetization. Choosing to invert for susceptibility will lead to grossly misleading inversion results. Instead, the correct choice is to invert for the magnetization vector (e.g. Ellis et al. 2012). To give some idea of the importance of choosing the correct model type in an exploration context, consider the inversion of TMI data acquired over the Osborne deposit (Rutherford et al. 2005), Queensland, Australia. Briefly, the Osborne deposit contains significant Cu-Au mineralization beneath 30-50m of deeply weathered cover, with current exploration focusing on mapping the high-grade steeply dipping mineralization to greater depths. TMI data were collected on a 40 m clearance with 40 m line spacing, and are shown in Figure 15.

Choosing to invert the Osborne TMI data for susceptibility yields the result shown in Figure 16, which shows a susceptibility distribution that is inconsistent with the known east-dipping mineralization, shown in black. Correctly choosing to invert for the magnetization vector yields the magnetization vector amplitude result shown in Figure 17, which is far more consistent with the geometry of the mineralization. This example illustrates that choosing the correct model element type is an important auxiliary input.
To this point we have addressed the auxiliary information implicit in the choice of model element size, geometry, type, and padding. We now turn to some of the more explicit auxiliary information used to solve the geophysical inverse problem, that is the weighting functions contained in the model objective function defined in Eq (1).

**Model Object Function based Auxiliary input**

We stated in the previous section that the main auxiliary input into the geophysical inverse problem enters through the model objective function, in Eq (1). This is because it contains the model weighting function which fundamentally controls the location of targets recovered by the inversion. To illustrate the dramatic effect that the weighting function has on an inversion, we show the result of four inversions of synthetic data over a simple buried prism with four different weighting functions. The model used to generate the data and the survey geometry are shown Figure 18. The actual type of survey (gravity, magnetics, etc.) is immaterial for this example. In Figure 19 we show section views of results of inversions with having different depth dependence (VOXI, 2012), increasingly forcing the recovered physical property deeper into the model, subject to the condition that predicted response still fits the original data. Of course, this is a simple manifestation of the non-uniqueness of geophysical inverse problems.
Figure 18: A simple model of a prism (magenta) buried in a uniform half-space with survey points (red.)

Figure 19: Clockwise from top left: a section through the model recovered by inversion using 's with different depth dependence producing: a surficial, a shallow, a medium, and a deep model. The location of the prism is shown faintly for reference.

The simple synthetic example of Figure 19 shows the importance of weighting in an academic sense. Next we show the effect of this auxiliary input on inversion of TMI field data from the San Nicolas deposit, Figure 2. Here we will apply the same 's as used in the prism example to produce a shallow, medium, and deep inversion result (Figure 20, Figure 21, Figure 22, respectively), a process we call "exploring the solution space". Fortunately we have a deposit model for San Nicolas (CAMIRO, 2004) which helps to give context to the inversion results. The ore zone is shown in red, with a nearby fault and rhyolite intrusive. The shallow, medium, and deep inversions all generate responses which fit the field data but yield very different geologic interpretations. It is only with the aid of an accompanying geologic model that a meaningful choice can be made. In this case, the geologic model is that the mineralization predominately overlies the rhyolitic lava domes and underlies the mafic extrusives. (Johnson et al. 2000). This suggests that the medium model is the appropriate solution. We emphasize that it is the combination of geologic model and geophysical inversion which yields an interpretation with value to the exploration process.
Figure 20: The inversion result for the shallow weighting function inversion is shown as the yellow thresholded voxel model. The ore zone from the deposit model is shown in red. A SW trending fault, a rhyolite intrusive, and bottom of the prospective sequence are shown in shades of blue.

Figure 21: The inversion result for the medium weighting function inversion is shown as the yellow thresholded voxel model. Compare with Figure 20.

Figure 22: The inversion result for the shallow weighting function inversion is shown as the yellow thresholded voxel model. Compare with Figure 20 and Figure 21.
We have seen how the use of a weighting function can be used to explore the geophysical solution space in the inverse problem. Now we consider a slightly different application of the model weighting function, that is, iterative reweighting inversion (IRI) focusing (VOXI, 2012b). Here we use the model weighting to take the standard smooth inversion and to compact the target into a smaller volume. This seemingly simple procedure can have dramatic effects on the quality of an inversion, which we demonstrate by applying the IRI-Focusing method to the Cannington TMI data shown in Figure 1. In Figure 23a we show the results of a standard smooth inversion, that is, where the weighting function causes a minimization of the first derivatives of the model. As expected with smooth inversion, the recovered model identifies some susceptibility in vicinity of the known mineralization however the result is rather vague and makes interpretation difficult. In contrast, using the IRI-Focusing weighting yields the result shown in Figure 23b, which clearly identifies the dip of the known mineralization as well as giving a better estimate of the susceptibility. These two improvements make interpretation far more reliable. Again we see that the weighting function is a very important auxiliary input in any geophysical inversion.

![Figure 23: Left: A section through the inversion of the Cannington data in Figure 1 using a standard smooth model weighting function. Right: The corresponding section from an inversion using a IRI-Focusing. Notice that the IRI-Focusing significantly improves the resolution of the target, and gives a much better representation of the true mineralization, approximated by the dark blue outline.](image)

**Conclusion**

The results from a geophysical inversion depend not only on the survey data and explicit geological constraints, but also on a large number of auxiliary inputs. In this work we address a subset of auxiliary inputs which the practicing geophysicist is likely to encounter. We saw that appropriate choice of:

1. data error level,
2. linear trend removal,
3. model element size, type, form, and padding, and,
4. weighting function,

all play an important role in determining whether an inversion produces an earth model which extracts the maximum amount of information from the data while not introducing spurious artefacts or a misleading interpretation. This leads to some practical considerations: first, software designed for inversion of geophysical data should provide the geophysicist with intelligent defaults for the auxiliary inputs; second, inversion software should be designed to allow the geophysicist to rapidly and efficiently work through a number of different auxiliary parameter settings to find the best settings for a particular
survey dataset; and third, for each dataset geophysicists should expect to run a number of inversions in an iterative process optimizing the auxiliary input. A single “blind” inversion may prove useful in early exploration and target evaluation but it is unlikely to produce optimal results. These three considerations would apply to any well-posed inverse problem, however, the geophysical inverse problem is ill-posed and requires an extra layer of auxiliary input in the form of a model weighting function (i.e. regularization). The correct choice of weighting function, and meaningful inversion outcome, depends on knowledge of the geologic expression of the target under investigation. Consequently geophysicists also require ability to easily generate appropriate weighting functions in their inversion software and to explore the space of geophysically viable physical property models.

It has long been accepted that producing meaningful exploration outcomes using geophysical inversion requires high quality survey data, and more recently it is increasingly believed that good geologic constraints are also required. The results presented in this work demonstrate that a good choice of auxiliary parameters forms the third component of the successful inversion triumvirate: good geophysical data, good geologic constraints, and good auxiliary inputs.

References

Ellis, R. G., MacLeod, I., N., de Wet, B., 2012, Inversion of magnetic data for remanent and induced sources, ASEG Extended Abstracts 2012(1) 1 – 4, Brisbane.

Ellis, R. G., MacLeod, I., N., 2013a, Constrained voxel inversion using the Cartesian cut cell method, ASEG Extended Abstracts 2013, Melbourne.


