Tilt Derivative Made Easy

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Theory

The tilt angle θ of a magnetic anomaly A is equivalent to local phase and given by

$$\theta = \tan^{-1}\left(\frac{\partial A/\partial z}{\partial A/\partial h}\right) \quad ,$$

where the numerator and denominator are vertical and horizontal derivatives of the anomaly, respectively, the latter given by

$$\partial A/\partial h = \sqrt{(\partial A/\partial x)^2 + (\partial A/\partial y)^2}$$

(Verduzo et al., 2004). The tilt derivative offers several advantages in the interpretation of magnetic anomalies. First, the dependence of θ on magnetization is the same in both the horizontal and vertical derivatives, so weakly magnetic bodies are treated with the same weight as strongly magnetic bodies. Second, θ has a very simple form over simple bodies. Over a vertical contact, for example,

$$\theta = \tan^{-1}(x/z), \tag{1}$$

where x is horizontal distance from the contact and z is depth (Salem et al., 2007). As noted by Salem et al. (2010), equation 1 provides a graphical means to estimate z from θ: noting that $\theta = \pi/4$ when $x = z$, we simply measure the horizontal distance between the zero and $\pi/4$ contours of θ, and this distance provides an estimate of z along the contour. Thus, the mapped shape of the zero contour indicates the mapped shape of the causative source, and the horizontal distance between the zero and $\pi/4$ contours provides an estimate of the depth of the body beneath the zero contour. Alternatively, we could measure the horizontal distance between the zero and $\pi/8$ contours to find $0.4142z$, or measure the horizontal distance between the zero and $\pi/16$ contours to find $0.1989z$, and so on. Using smaller contour intervals provides better estimates of z. Although somewhat cumbersome, horizontal distances can be determined either graphically or with mapping algorithms, such as those found in ArcGIS, Oasis montaj, or other GIS utilities.

Each of the graphical measures described above is equivalent to estimating the horizontal gradient of θ at $x = 0$. From equation 1, the horizontal gradient is given by

$$d\theta/dh = z/(x^2+z^2),$$

and, at $x = 0$,

$$z = [d\theta/dh]^{-1}\,. \qquad (2)$$

Equation 2 provides a very simple way to map both the edges and depths of causative sources from reduced-to-pole magnetic anomalies without the need for graphical methods: We simply map the values of $[d\theta/dh]^{-1}$ along the zero contour of θ.

Implementation in Oasis montaj

In Oasis montaj, a map-based interpretation of the tilt derivative is a matter of just a few steps. Starting with a grid of reduced-to-pole magnetic anomaly values, we do the following:

1. Calculate the tilt derivative using the standard Oasis montaj GX. By default, the tilt-derivative GX provides both the tilt derivative and its horizontal gradient. (`Magmap → Tilt Derivative`)
2. Map the zero contour of the tilt derivative without labels. (`Map Tools → Contour`)
3. Export the zero contour layer to a shapefile. (`Map → Export`)
4. Import the shapefile back into a Geosoft database. Specify "New database with shape database(s)". The zero contour will be represented in the shape database as X and Y channels. (`Map → Import`).
5. Determine the value of the horizontal gradient at each x,y coordinate, thereby creating another channel. (`Grid and Image → Utilities → Sample a Grid`)
6. Add a channel to the database representing the negative of the reciprocal of horizontal gradient. (`Database Tools → Channel Math)`
7. Tidy up the database as desired, decimating points based on X and Y and windowing points based on depth.
8. Use colored symbols to plot the value of depth at each xy coordinate. (*`Map Tools →`* `Symbols → Zone Colored`)

Examples

Figure 1 describes a synthetic test of the method described above. The total-field anomaly caused by two rectangular prisms is shown in Figure 1A. Both prisms have vertical sides, and magnetization and ambient field are both vertical; i.e., the anomaly is reduced to the pole. The northern prism is at a depth of 1 km; the southern prism at a depth of 2.5 km. Figure 2 shows the outline of the prisms and estimated depths, as determined by the tilt-derivative method.

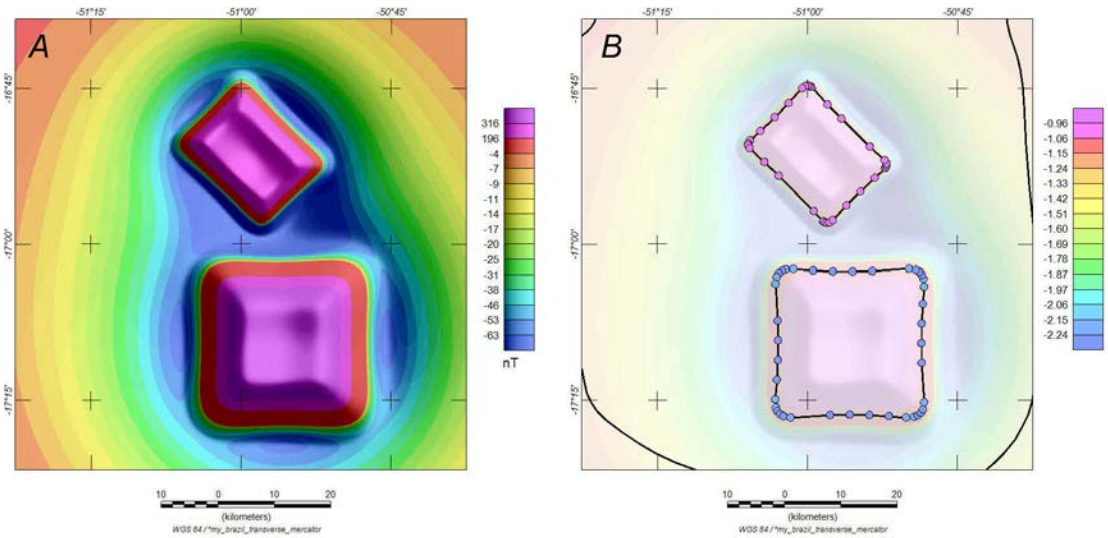

Figure 1. Synthetic test of the tilt-derivative method. (A) total-field anomaly over two vertical-sided prisms. The northern prism is at a depth of 1 km; the southern prism is at a depth of 2.5 km. Magnetization and ambient field are vertical. (B) Shape and depth of the two prisms as estimated with the tilt-derivative method.

Figure 2 shows an application to real data. Total-field anomalies (Fig. 2A) are from north-central Washington, north of Wenatchee (Blakely et al., 2012). Short-wavelength anomalies in the southeast quadrant are caused by strongly magnetic rocks of the Miocene Columbia River Basalt Group (CRBG) exposed in this area. Long-wavelength anomalies elsewhere are caused by pre-CRBG intrusive rocks, continental sediments, and metamorphic rocks, essentially forming the magnetic basement throughout the area (Blakely et al., 2012). Figure 2B shows an interpretation based on the tilt-derivative.

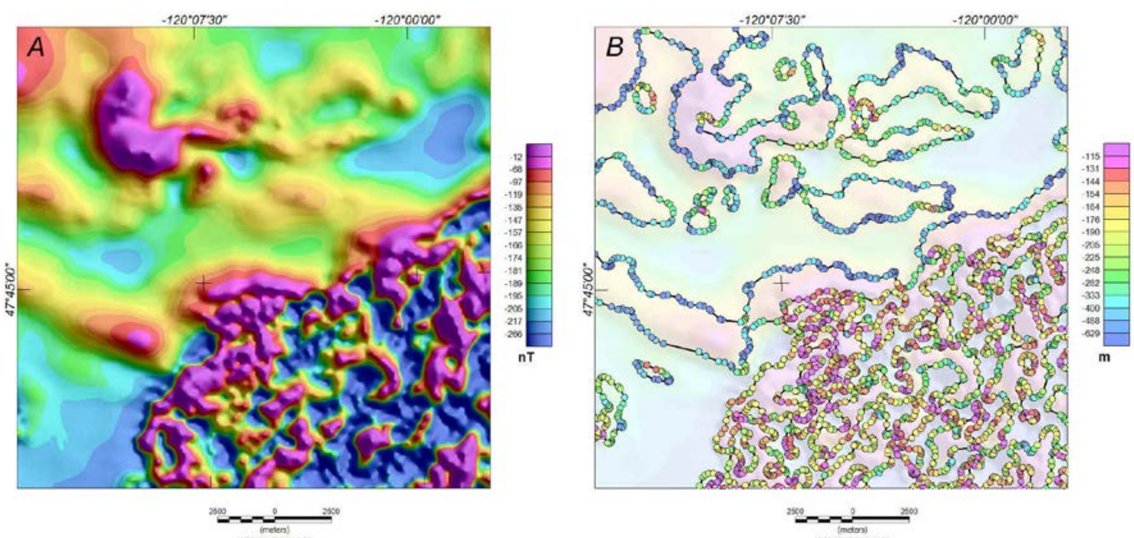

Figure 2. Figure 2. (A) Total-field magnetic anomaly from north-central Washington State. (B) Source outlines and depths estimated from tilt derivative. Black lines, zero-contour of the tilt derivative; colored dots, depths estimated from the reciprocal of the horizontal gradient of the tilt derivative.

References

Blakely, R.J., Sherrod, B.L., Wells, R.E., and Weaver, C.S., 2012, Peeking below Columbia River flood basalts with high-resolution aeromagnetic data: implications for central Washington earthquake hazards [abstract]: Abstract GP13C-02, presented at 2012 Fall Meeting, AGU, San Francisco, California, December 3-7, 2012.

Salem, A., Williams, S., Fairhead, J.D., Ravat, D., and Smith, R., 2007, Tilt-depth method—a simple depth estimation method using first-order magnetic derivatives: The Leading Edge, v. 26, p. 1502-1505.

Salem, A., Williams, S., Samson, Esuene, Fairhead, D., Ravat, D., and Blakely, R.J., 2010, Sedimentary basins reconnaissance using the magnetic tilt-depth method: Exploration Geophysics, v. 41, p. 198-209.

Verduzco, B., Fairhead, J.D., Green, C.M., and MacKenzie, C., 2004, New insights into magnetic derivatives for structural mapping: The Leading Edge, v. 23, p. 116-119.