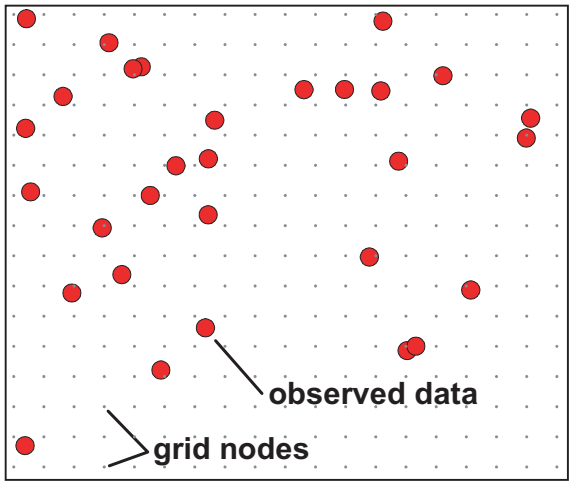
# Topics in Gridding



## Overview

When dealing with two-dimensional data, it is useful to represent the data by determining its value at points located equally far apart at the nodes of a grid, as shown in the following figure:




Data in grid format is suitable for a number of two-dimensional processes, such as image processing and two-dimensional filtering.

The values at the grid nodes can be determined by taking readings at the node locations. However, in practice this is seldom convenient and it is more likely to have data that has been collected at random locations, or which has been collected at a relatively high sample density along more widely separated parallel lines. Such raw data is commonly referred to as ***XYZ*** data because each data point has an (x,y) location and one or more measured (z) values. The process of ***gridding*** takes XYZ data and interpolates the readings to determine the values at the nodes of a grid. The resultant grid data set is known as a ***grid***.

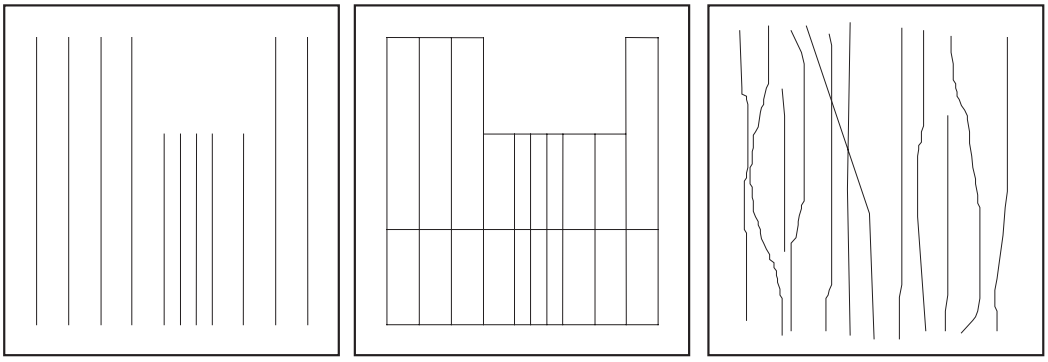
Geosoft provides three programs that interpolate raw XYZ data to produce a grid:

1. **RANGRID** interpolates the data by fitting a two-dimensional surface to the raw XYZ data in such a way that the curvature of the surface is minimized. **RANGRID** is ideal when data is sparsely sampled and the surface is expected to be relatively smooth or continuous between data points.
2. **KRIGRID** interpolates the data using kriging. Kriging is a geostatistical method that determines the most probable value at each grid node based on a statistical analysis of the entire data set. Based on Kriging statistics, **KRIGRID** is also able to produce an error grid, which gives an indication of the degree of confidence at each grid node. **KRIGRID** is ideally suited to geochemical or other geological sample-based data (as opposed to geophysical data). Because kriging can be very slow, the size of a data set may be a limiting factor in choosing to use **KRIGRID**.

3. **BIGRID** is a bi-directional gridding program designed to rapidly interpolate roughly parallel line-based data. **BIGRID** uses linear, minimum curvature or Akima splines to interpolate grid nodes between lines in the direction of the overall trend of the data, which is usually perpendicular to the survey lines. **BIGRID** can be 10 to 100 times faster than **RANGRID**, and up to 1000 times faster than **KRIGRID**. It is the only program that can take advantage of a strong line-to-line correlation of otherwise narrow features in line data. **BIGRID** cannot be applied to randomly distributed XYZ data. Line data that is measured along orthogonal lines is also not well suited to **BIGRID**.

## RANGRID, BIGRID or KRIGRID?

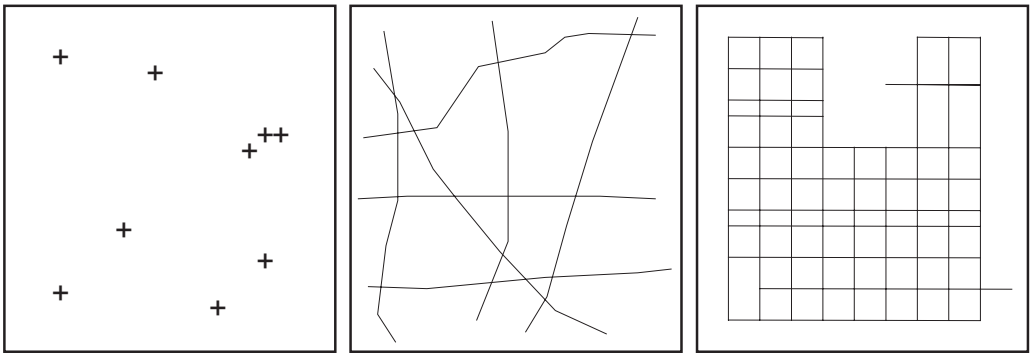
Use **BIGRID** if the data is collected along lines that are roughly parallel, as in the following examples:



**Parallel Lines** | **Lines with some tie lines** | **Roughly Parallel Lines**

**BIGRID** is ideal in these situations, especially if there is a high sample density down the lines relative to the line separation. Furthermore, **BIGRID** is able to join narrow features that extend from line to line perpendicular to the line direction. Note that in the middle example, **BIGRID** is not able to use the tie lines because of the way the gridding algorithm works. If the data on the tie lines is important, **RANGRID** or **KRIGRID** should be used.

Use **RANGRID** or **KRIGRID** when the XYZ data is ***not*** sampled along lines that run in roughly the same direction. Such data are often called Random because they give a random appearance when the data locations are plotted. Also, line data in which survey lines are orthogonal (or have 'random' directions) should be gridded with **RANGRID** or **KRIGRID**. The following figure illustrates these types of data:



**Random** | **Non-parallel Lines** | **Orthogonal Lines**

If the data is relatively smooth between sample points or survey lines, **RANGRID** should be used. If the data may be variable between sample locations, or is known to be statistical in nature (such as geochemical data), is poorly sampled or clustered, use **KRIGRID**.

## The Nyquist frequency and aliasing

The terms "aliasing" and "Nyquist frequency" are often used to describe problems and limitations in gridded data sets. Anyone who creates gridded data, or who works with gridded data should understand what these terms mean and their importance in gridding.

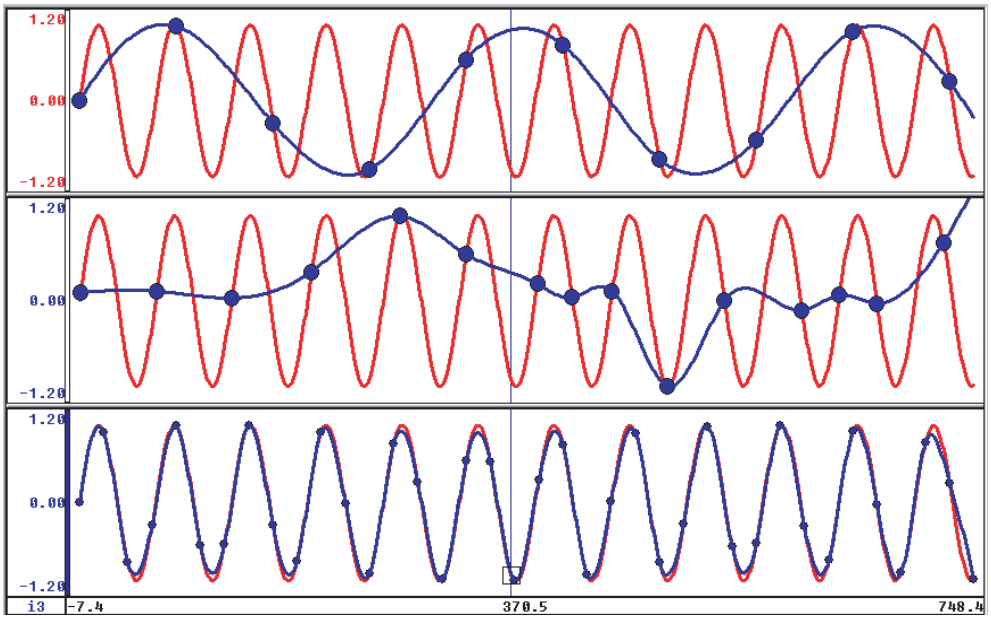
The Nyquist frequency is the highest frequency (shortest wavelength) that it is possible to measure given a fixed sample interval. It is defined by the expression

$$N = \frac{1}{2d}$$

where $d$ is the sample interval.

Gridded data sets are limited in the detail of information that they contain by the Nyquist frequency (1/(2*cell size)). Original survey data is limited in the detail of information that has been measured by 1/(2*nominal sample interval). Line-based surveys are limited in their down-line detail by 1/(2*sample interval) and in their across-line information by 1/(2*line separation). In other words, if it has not been measured, we can't see it.

Aliasing results when the actual data that is being sampled contains significant information that has a shorter wavelength (higher frequency) than the Nyquist. For example, in line-based surveys, we often see short-wavelength features along lines that are too poorly sampled across lines to be properly represented in a grid. The result is a "noisy" appearance in the grid that can be seen between survey lines. A important problem with aliasing phenomena is that you often cannot see the aliasing because it can be hidden by the chance locations of the samples. Inversely, the chance locations of samples can give the appearance of much longer wavelength features than really exist.




THIS FIGURE ILLUSTRATES THE PROBLEM OF ALIASING THAT CAN RESULT FROM INSUFFICIENT SAMPLING OF INFORMATION. IN THIS CASE THE DATA BEING SAMPLED IS A SIMPLE SINE. IN THE TOP PROFILE, AN EVEN SAMPLING OF THE DATA IMPLIES A SINE FUNCTION, BUT AT THE WRONG WAVELENGTH. THE MIDDLE PROFILE SHOWS A MORE RANDOM SAMPLING, AND THE IMPRESSION IF ANOMALOUS HIGH AND LOW AREAS IS COMPLETELY FALSE. THE BOTTOM PROFILE SHOWS THE RESULT OF SAMPLING AT JUST UNDER THE NYQUIST FREQUENCY, WHICH PRODUCES ALMOST THE CORRECT RESULT.

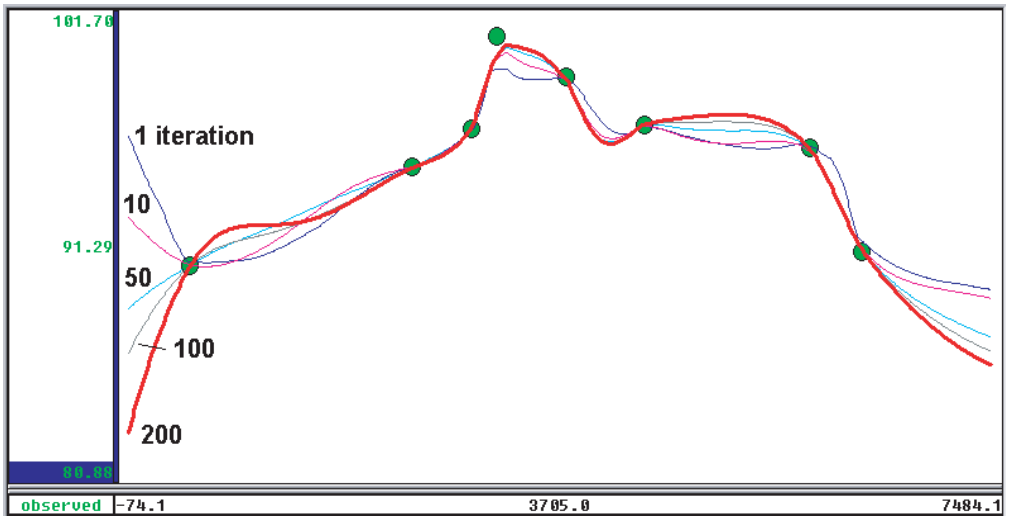
The Nyquist frequency and the effects of aliasing must be considered at a number of stages in the creation and use of gridded data:

1. The design of surveys should take into account the expected spatial size of information that is measured, and the detail required for the data to be useful. Note that because of aliasing, it may be necessary to sample to a higher density than is required for the interpretation of the data. This allows you to remove aliased noise from the data as part of the gridding process (if it isn't measured, you can't remove it either!).
2. Pre-gridding filters should be applied to data to remove wavelengths that may be aliased at the chosen grid cell size.
3. The grid cell size must be chosen to sample the required detail in the data.
4. When working with gridded data, you cannot see detail smaller than the grid cell size.

## Minimum curvature gridding with RANGRID

**RANGRID** fits a minimum curvature surface to the data points using a method similar to that described by Swain (1976) and Briggs (1974). A minimum curvature surface is the smoothest possible surface that will fit the given data values.

**RANGRID** first estimates grid values at the nodes of a coarse grid (usually 8 times the final grid cell size) based on the inverse distance average of the actual data within a specified search radius. If there is no data within that radius, the average of all data points in the grid is used. An iterative method is then employed to adjust the grid to fit the actual data points nearest the coarse grid nodes. Once an acceptable fit is achieved, the coarse cell size is divided by 2 and the same process is repeated using the coarse grid as the starting surface. This is process repeated until the minimum curvature surface is fit at the final grid cell size.




### Cell Size Selection

Grid cell size should not be much less than half the nominal data point interval found in the areas of interest. If a cell size is too small it will result in **RANGRID** consuming more processing time than necessary. A small cell size may also require a reduction in the iteration tolerance and an increase in the number of iterations to achieve an acceptable result.

Clustered data can also be gridded to a coarser grid cell size that would normally be desired. This will naturally de-alias the data because of the wider grid cell size. The grid can be re-gridded afterwards to a smaller cell size.

## Controlling Grid Quality

The RANGRID algorithm is iterative. This means that the modelled surface approaches the ideal minimum curvature surface in steps. The following figure shows a profiles for a different numbers of iterations through a set of observed points.

Note that the modelled surface will only reach the true minimum curvature surface after an infinite number of iterations, which is impractical. You control when to stop iteration through the use of three controls:

- tolerance
- percentage of points required within the tolerance
- maximum iterations

Iterations at any coarseness level will stop when the required percentage of points are within the tolerance of the true value, or when the maximum iterations have been reached. By default, the tolerance is set to 1% of the data range, the percentage is set to 99%, and the maximum iteration value is set to 100. We have found these defaults to be acceptable in most situations. However, if you are dealing with a mixture or sparse and detailed data, or if you intend to extract short wavelength information from the grid, you may need to force more iterations to be applied to the data. To do this:

1. Set a smaller tolerance in order to force more iterations.
2. Increase the percentage of points that must pass the tolerance test. This will normally only make a difference when you have a few points or areas in the data set that are not being honored by the grid.
3. Increase the maximum number of iterations if the current maximum is limiting the iterations and even more iterations are required. Note that 100 iterations should be sufficient in most cases and 200 iterations are considered extreme.

## De-aliasing clustered data

Clustered data, or data with areas of small sample interval separated by larger undersample areas, can cause the minimum curvature surface to have undesirable overshoots. If these are observed in your data, the following techniques can help to minimise the problem.

1. Increase horizontal tension. This will have the effect of stretching the surface over the observed points and minimise over-shoot. However, this can create undesirable 'sharpness' around observed points.
2. Increase the low-pass de-sampling factor, which is the number of grid cells over which average data points before gridding. This effectively removes higherfrequency detail that may be causing over-shoots.
3. Grid to a larger cell size, then re-grid the grid to the desired working cell size. This is similar to the low-pass de-sampling technique since the data will be averaged for the new cell size, but the re-gridding process can be much faster, and the effect of the coarser gridding staged in RANGRID will be more important in the final result.
4. Set the blanking distance to not include the over-shoot areas of the grid. The blanking distance is the maximum distance from actual data points to be included in the final grid.

## Strengths

RANGRID can work with any number of data points in any location. Data does not have to be organized as lines.

Minimum curvature gridding does not impose a directional bias in the data, and it is best for data that does not have a dominant trend direction. This is the principle strength of minimum curvature gridding over bi-directional gridding of line data.

Minimum curvature gridding can include ties line information in the gridding process. Note that tie lines must be properly levelled to the survey lines.

### Weaknesses

Minimum curvature gridding does attempt to find the minimum curvature surface, but the resulting surface is not a true minimum curvature surface. Because of the way the iterative process works, the final grid surface will be closest to the required surface at the original observation locations. This produces a 'pimpling' effect in filtered products that enhance the high-frequency parts of the data.

When gridding line data, minimum curvature gridding is not able to strengthen trends perpendicular to the survey lines. This can result in a 'bulls-eye' or 'string-of-beads' effect for features that have a narrow width with respect to the survey line direction.

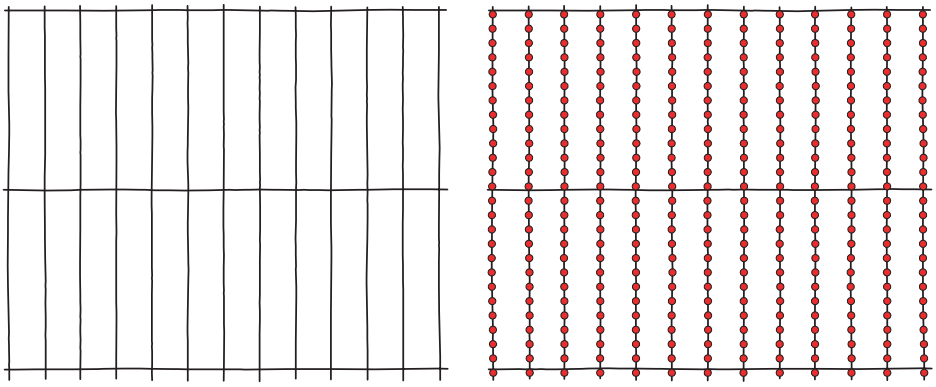
Clustered data also present a challenge to minimum curvature gridding. The minimum curvature surface in highly sample parts of a clustered data set can produce undesirable highs or lows in the more poorly sampled parts of the data. Increasing horizontal tension can help to control this problem, but the resulting grid will have more curvature around data points. You can also apply a low-pass de-sampling factor to de-sample-the data to a coarser separation before gridding. This will remove the effect of shorter-wavelength features in the clustered areas.

RANGRID can be quite slow for very large data sets.

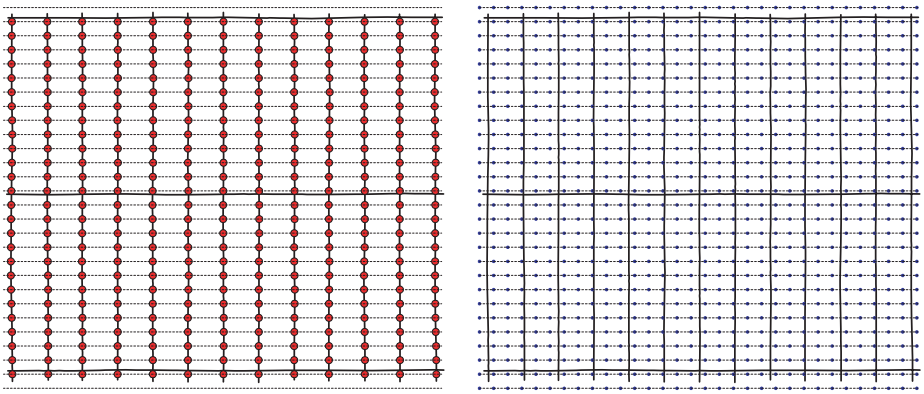
## Bi-directional gridding of line data with BIGRID

The bi-directional method of gridding, which is used in **BIGRID**, is ideal for line oriented data because it inherently tends to strengthen trends perpendicular to the direction of the survey lines.

The gridding process is carried out in two principle steps:



1. Each survey line is interpolated along the original survey path to yield data values at the intersection of each required grid line. Note that horizontal tie lines cannot be used in this process because they do not intersect the horizontal grid lines (or vise-versa for vertical tie lines, horizontal survey lines).



2. The intersected points from each line are then interpolated in the across-line grid direction to produce a value at each required grid point. The second pass of interpolation creates **grid lines**. A grid line is a series of numbers that represent all the values along a single grid row.

Geological trends in the data can be emphasized by the appropriate orientation of the grid so that the second interpolation is in the direction of strike. In addition to trend enhancement, **BIGRID** allows the method of interpolation to be selected independently for the down-line and across-line directions. The interpolations available are linear, cubic spline (minimum curvature) or Akima spline.

Filtering of the line data before interpolation is also possible. **BIGRID** can design and apply non-linear and/or linear numerical filters to the original line data. The use of the non-linear filter is a very effective way to remove data spikes (undesired highamplitude short-wavelength features) from the original data.

### Cell Size Selection

By default, BIGRID will choose a grid cell size that is roughly one-quarter the line separation. However, BIGRID can have difficulty determining an accurate line separation, and we recommend that you always choose an appropriate grid cell size.

The grid cell size should not be smaller than one eighth the nominal line separation. If the cell size is unreasonably small, a short-wavelength error that appears as lines perpendicular to the line direction can result, especially if there is noise in the data.

### Low-pass de-aliasing filter

BIGRID will preserve and honor all wavelength information up to the grid cell size. Unwanted short-wavelength features can be removed by the application of a low-pass filter, which can be specified as part of the gridding process. Note that the relatively wide survey line spacing rather than the sample spacing down lines normally limits the Nyquist frequency for the data. The application of a low-pass filter to remove wavelengths one half to one quarter the Nyquist wavelength defined by the line separation may be required.

### Strengths

BIGRID main strength is performance of very large data sets. The algorithm is very efficient relative to the other gridding methods.

Unlike minimum-curvature gridding, the gridded surface produced by BIGRID will honor the data exactly, and the interpolation splines are exact.

BIGRID is able to enhance trends in the gridding direction, which can be very important in geologically linear areas.

### Weaknesses

BIGRID's ability to enhance trends is also its weakness when the features in the data have short wavelength along lines, but do not run in the grid trend direction. Such features will produce ellipsoids and ellipsoidal 'beads', just as RANGRID produces 'bulls-eyes'.

Bi-directional gridding cannot work with tie-lines, nor can it work with randomly located data.

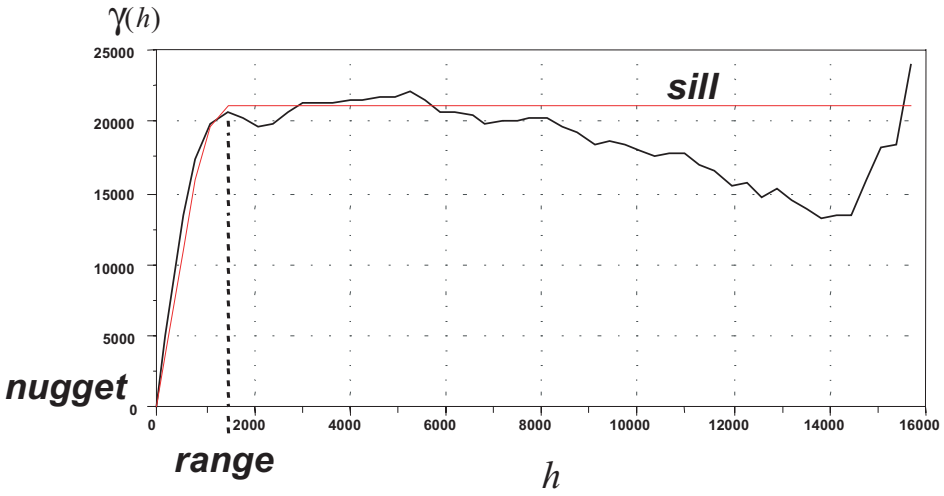
## Statistical gridding with KRIGRID

In contrast to RANGRID, which attempts to fit the smoothest possible *surface* to the data, KRIGRID uses a statistical analysis of the data to *predict* the values at each grid node based on maximum probability (minimum error). Because of this, KRIGRID is ideally suited to data sets that have a statistical basis, such as geochemical and geological sample data. KRIGRID is rarely used with geophysical data, which tends to follow a natural smooth surface.

A fundamental step in Kriging is the creation of a Variogram, which is a graphical measure of the statistical relationship of the sample data as a function of distance (h): two dimensions.

$$\gamma(h) = \frac{\sum [Z(x+h) - Z(x)]^2}{2}$$

The variogram is calculated by going through every pair of data points, evaluating this function and averaging values at the same distance. Below is a sample Variogram of a line-based airborne magnetic survey:




The primary characteristic to note is the natural increase in variability as the distance between data points (*h*) increases. Also, at a certain distance (called the ***range***), the variogram becomes flat at a level called the ***sill***. Up to the *range*, the data is correlated as a function of distance. Beyond the range, the data is not statistically correlated except that we only know that values should naturally lie within a maximum envelope.

A third parameter used to describe the Variogram is the ***nugget***. This is the point at which the variogram model intersects the (d=0) axis. The *nugget* represents the measurement error, or repeatability of the survey measurements.

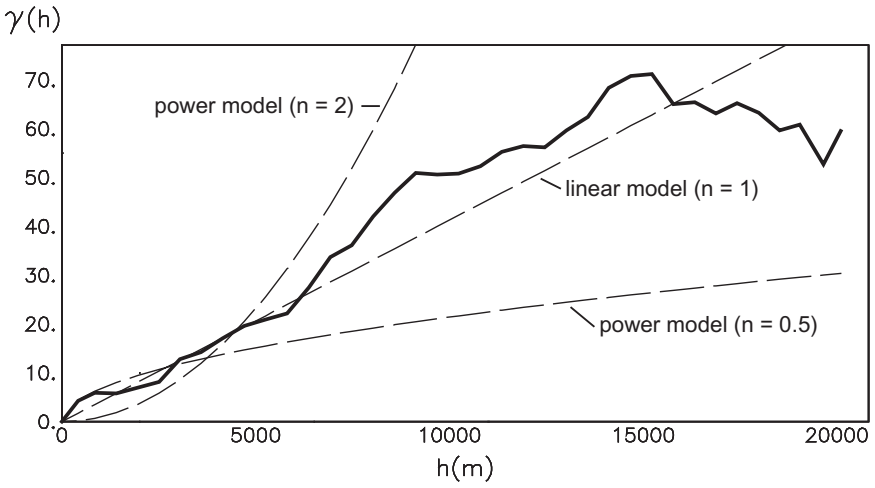
In order to perform Kriging, a model of the ideal variogram that is appropriate for the data is required. Kriging in OASIS montaj™ provides a number of basic model types that can be used. It is important to note the difference between an observed variogram and the model variogram. The observed variogram is only used as the basis for choosing an appropriate model variogram. The model variogram is the assumed ideal variability given an infinite number of samples in an ideal model.

### Power Model

The power model is the simplest model, but it is unable to effectively model data that contains a sill. However, the linear model (power = 1) can accurately model the linearity of the variogram between the nugget and the range. A linear behavior in the variogram at the origin is the most common in mining geological applications. If the nominal sample interval of the data is less than the range, the linear model is often suitable. The linear model is the only model that OASIS montaj™ can calculate automatically from the observed variogram. The other models require you to evaluate the variogram and set the parameters manually.

The power model is used by default if no model is specified.

$$\gamma(h) = nugget + (slope)h^n$$

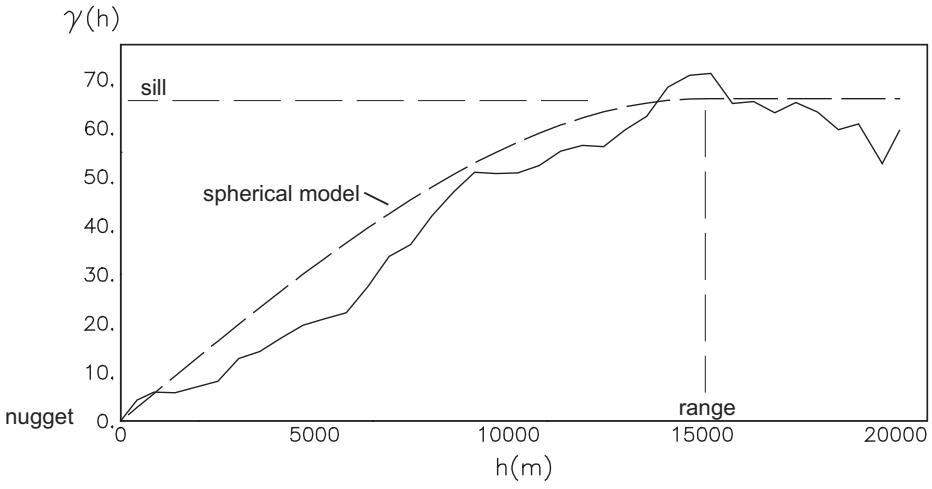



### Spherical Model

The spherical model is the most common model used for geological data. It can accurately model linearity at the origin, and it accounts for a sill. However, you must estimate the nugget, range and sill from the observed variogram.

$$\lambda(h), (h < range) = nugget + \frac{1.5h(sill - nugget)}{range - 0.5\left[\frac{h}{range}\right]^3}$$

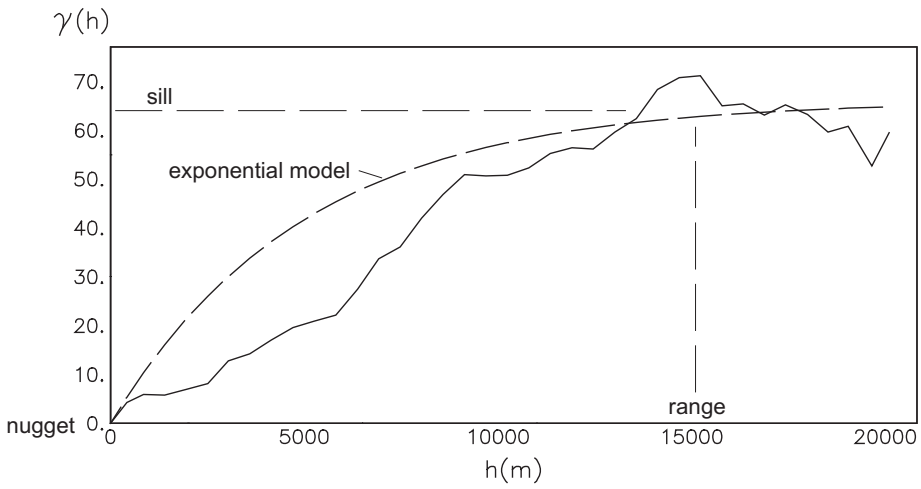
$$\gamma(h), (h >= range) = sill$$



## Exponential Model

The exponential can also effectively model a sill, but the shape of the model between the nugget and the range has a more gradual transition.

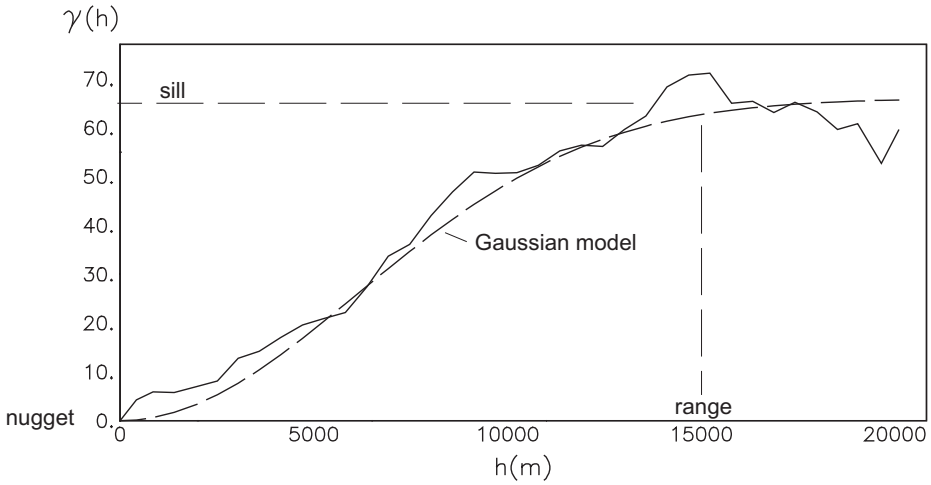
$$\gamma(h) = 1 - \exp(-h / range)$$




## Gaussian Model

The Gaussian model is able to model parabolic behavior at the origin, although this is seldom found in geological applications. With this model the sill is actually never reached, and at the range the model is 5% less than the sill. Like the spherical model, you must specify the nugget, range and sill to use the Gaussian model.

$$\gamma(h) = 1 - \exp(-r^2 / range^2)$$




## Kriging Error Grid

One benefit of Kriging is that it can create a grid of the standard deviation of the estimate at each grid point. Based on statistical theory, we have 68% confidence that the estimation is correct to ±(1 standard deviation), and 95% confidence that the estimation is correct to ±(2 standard deviations).

### Grid Cell Size

The KRIGING algorithm is not sensitive to cell size. The cell size should be chosen to adequately sample the information of interest. This is normally one-half to onequarter the nominal sample interval.

### Practical Notes

The majority of geophysical data sets are statistically under-sampled, which means that the benefit of accurate statistical estimates is diminished. The under-sampling is most easily seen by looking at the range, is very often just beyond the nominal sample interval of the data. Because of this, it is rarely worth the effort to accurately model the variogram for a given data set. The default linear model usually produces the same result as a carefully modeled variogram.

The main application for Kriging is for gridding geochemical data, where statistical relationships are more common. However, as with geophysical data, geochemical data is rarely sampled to a sufficient density to justify the effort of careful modelling of variograms.

Kriging can produce better visual results than RANGRID for clustered data. Clustered data has regions of very dense observations together with wide regions with few observations. In this situation RANGRID will often produce undesirable high and low perturbations in the under-sample areas. The statistical nature of Kriging can reduce this effect.

Kriging is also the slowest gridding technique, especially when there are many data points. It works best with less than 500 data points, in which case the entire data set is solved in a single matrix inversion. If there are more than 500 data points, a moving matrix is used to calculate the prediction at each grid point. This is slower and can lead to edge effects at the limits of the smaller matrix window.

Logarithmically distributed data should be gridded using the log gridding option. This will grid the log of the data in log space, then convert the gridded result back to linear space. This also prevents the creation of negative values in the grid.

## Trend enhancement (new application – TRENDDB)

The main challenge (and weakness) of minimum curvature gridding and bidirectional gridding is the relatively high sample interval down survey lines relative to the much wider line separation. The high sample interval down lines allows us to sample very detailed features perpendicular to the line direction. However, the much poorer sampling across lines is insufficient for gridding algorithms to interpolate accurately.

As part of the Advanced Grid Utilities Package in OASIS montaj™ 4.3, we are releasing a new technique designed to find such trends in line data. The method locates all local maxima and minima along each survey line, then creates a triangulated mesh of both the maxima and minima points. The meshes are decomposed into trend line segments that do not cross and produce a maximum linearity in a preferred direction. These 'trend lines' are added to the data, which can then be gridded normally using any of the gridding programs. The trend lines will force the interpolations between lines to honor the defined trends.

As part of this workshop we are providing a preview copy of the trend gridding GX's. Note that the use of the Trend Gridding GX's requires the Advanced Grid Utilities license. The package includes:

| | |
|---|---|
| **`Trend_grid.omn`** | trend gridding menu |
| **`Trenddb.gx`** | finds trends in the data |
| **`Trenddigi.gx`** | interactive digitizing of trend lines |
| **`Trendpath.gx`** | plots trend lines on a map |

The principal GX in this suite is the TRENDDB GX. Following are the user parameters used by the GX.

### Data Channel for trends

The channel to find trends for.

### Window for Max-Min Search

All maxima and minima found are guaranteed to be the lowest or highest values within half this distance on either side of selected point. The distance is determined from the X and Y channel positions, measured cumulatively from point to point.

### Preferred angle for trends

The angile is the angle measured counter-clockwise from the horizontal (X-axis) in degrees.

### Allowable Deviation in angle

This is the amount by which the trend angle of individual trend sections may deviate from the preferred angle. For instance, if the preferred angle were 30 degrees, an allowable deviation of 30 degrees would search for trends at angle from 0 to 60 degrees.

### Maximum length for joins

It is usually good to set this to about twice the average line spacing. Too large a value may create trend lines across large “empty” regions, and can increase the processing time. The value should exceed the average line spacing, or no joining or extrema may occur.

### Maximum deflection for joins

This controls the local “straightness” of joins. The deflection is the length of the perpendicular dropped from the third point in a sequence to the line which joins the first two points. If the three points are co-linear, this value will be zero. If the input value is left blank, no check will be made for this condition.

### Minimum Length

Ignore all trend lines shorter than this. This test is applied after any resampling, and before breaking of lines at the breaking angle. If left blank, this test is not performed.

### Resampling Interval

The line may be splined at a fixed interval. For maximum effectiveness, this should be set to a value which is a fraction of the average line spacing; this could be the along-line sample interval. This will reinforce the data values along the trend lines. The start and end points are preserved. If left blank, this test is not performed.

### Breaking Angle

Some routines such as BIGRID require that lines increase monotonically in certain directions. Because trend lines are often nearly parallel to the first spline direction in BIGRID, they can be multiple-valued functions. Forcing breaks when the line direction switches "back and forth" over the first spline direction restores the monotonic nature to the all the lines, and ensures the entire trend line is used. The first spline direction is set in BIGRID using the “Trend Angle” parameter. (If it is left blank, this will be either 0 or 90 degrees, depending on the general flight line direction. ) Set the breaking angle to the Trend Angle for best results. For example: If flight lines are mostly vertical, then the initial BIGRID interpolation will be along horizontal lines. To ensure trend lines will be fully used, set the break angle to 0 degrees.

If the break angle is left blank, this test is not performed, and some trend information may not be used.

## Gridding References

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# Analytic Signal





## Overview

Nabighian (1972, 1984) developed the notion of 2-D analytic signal, or energy envelope, of magnetic anomalies. Roest, et al (1992), showed that the amplitude (absolute value) of the 3-D analytic signal at location (x,y) can be easily derived from the three orthogonal gradients of the total magnetic field using the expression

$$|A(x,y)| = \sqrt{\left(\frac{dT}{dx}\right)^2 + \left(\frac{dT}{dy}\right)^2 + \left(\frac{dT}{dz}\right)^2} \quad (3)$$

where

$|A(x,y)|$ is the amplitude of the analytic signal at $(x,y)$

$T$ is the observed magnetic field at $(x,y)$.

From this expression you can see that the analytic signal is a direct measure of the total gradient of the magnetic field. This is naturally a function of the distance to the magnetic source and the intensity of magnetization. In other words, as a rule, the closer the source and the greater the anomalous field, the greater the analytic signal. The analytic signal anomaly over a 2-D magnetic contact located at ($x$=0) and at depth $h$ is described by the expression (after Nabighian, 1972)

$$|A(x,y)| = \alpha \frac{1}{\left(h^2 + x^2\right)^{1/2}} \quad (4)$$

where $\alpha$ is the amplitude factor

$$\alpha = 2M \sin d(1 - \cos^2(I)\sin^2(A)) \quad (5)$$

and

$h$ is the depth to the top of the contact

$M$ is the strength of magnetization

$d$ is the dip of the contact

$I$ is the inclination of the magnetization vector

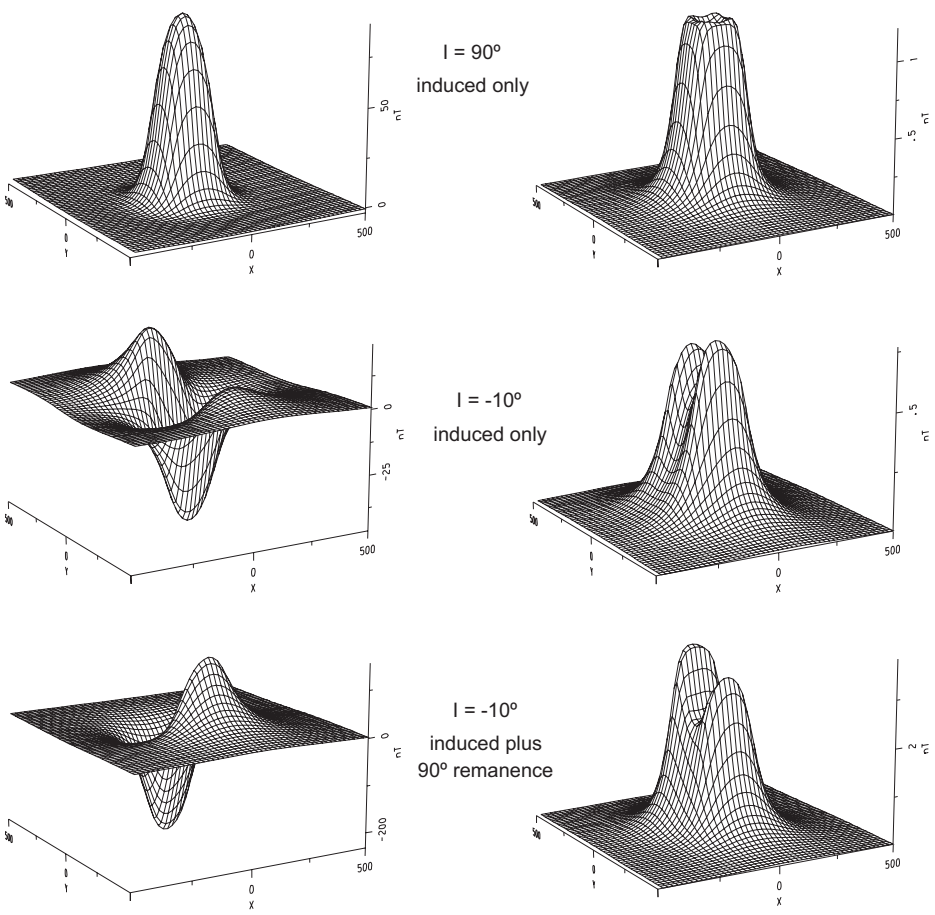
$A$ is the direction of the magnetization vector

Similarly, it can be shown that the analytic signal anomaly over a 2-D magnetic sheet (or dyke) is described by the expression

$$|A(x)| = \alpha \frac{1}{\left(h^2 + x^2\right)} \quad (6)$$

and over a cylinder

$$|A(x)| = \alpha \frac{2}{\left(h^2 + x^2\right)^{\frac{3}{2}}} \quad (7)$$




The analytic signal anomaly in all cases is a simple bell shaped function in which all directional terms are contained in the amplitude factor $\alpha$, which is a constant. Therefore, only the amplitude of the analytic signal is affected by the vector components of the model. The shape of the analytic signal depends only on depth. It is this characteristic of the analytic signal that makes it very useful for interpreting potential field anomalies. This also makes analytic signal a practical alternative to reduction-to-the-pole (RTP) when the goal is to simplify magnetic anomalies.

This shows the results of analytic signal processing over a simple prism model at different magnetic latitudes and with different remnant magnetization. The use of a 3- D perspective presentation is to clearly show the how the amplitude of the analytic signal peaks over the edges of the model. Note that the amplitude of the peaks is proportional to the magnetization at that edge as defined by equation (5). In this case, the prism width is four times the depth. For widths less than the depth, the peaks of the analytic signal will merge.

## Application in regional interpretation

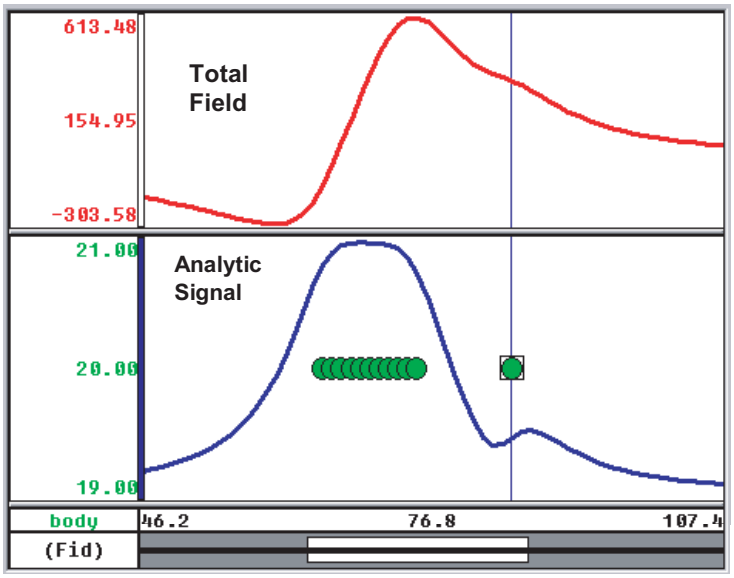
For regional geologic interpretation, an Analytic Signal map can be though of a approximation of the distribution of magnetic minerals in the ground, and in fact Analytic Signal maps look very similar to gridded geochemical maps. Areas of Analytic Signal highs will be located over more highly magnetic rocks.

## Application in anomaly source location

For detailed single anomaly interpretation, analytic signal is useful to verify the location of a source magnetic body. This is especially helpful when magnetic source bodies have unknown remanent magnetization.

However, interpreting location from Analytic Signal does require relatively good separation of interfering anomalies. If the anomaly in question is very strong relative to potentially interfering anomalies, the analytic signal will be accurate. If the anomaly is relatively weaker than interfering anomalies, or is on the flank of another anomaly, the peak of the analytic signal can be moved up to one-half wavelength from the true body location. This is normally easy to recognize by both the shape of the Analytic Signal, which will have an asymmetrically sharper low on the true location side, and by the fact that there is a strong neighboring anomaly.

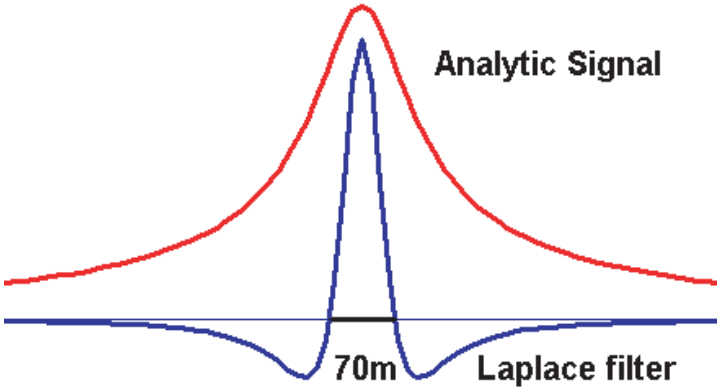
This is illustrated in the following example created from synthetic data. The circles indicate the location of a wide (left) and narrow (right) vertically dipping magnetic bodies.




### Depth to source interpretation

On clearly separated Analytic Signal anomalies, the width of the analytic signal at the inflation points is a measure of the depth to the source (MacLeod, 1993). A **Laplace** filter can be applied to the data, either in 1-D (-1,2,-1) or 2-D (0,-1,0,-1,4,-1,0,-1,0), to provide a visual measure of the distance between inflection points.

The following figure illustrated the interpretation of depth from the application of a Laplace filter to a profile over a contact. The distance between inflection points is approximately 70 metres, and the depth to the source would be (70/1.41), or approximately 50 metres. If the source were interpreted as a thin sheet, the depth interpretation would be 70 metres (70/1). Depending on the quality of the anomaly and the affect of interfering anomalies, you will have some confidence that the source lies somewhere between these limits.



## Limitations

Because analytic signal is calculated from derivatives, it is sensitive to shortwavelength noise in the data. This can be removed by simple low-pass filtering, or by vertical integration (MacLeod, 1992).

As mentioned previously, anomaly location can be complicated by overlapping or interfering anomalies. This should be addressed by care in the interpretation of locations, or by calculating the analytic signal on higher-order vertical derivatives second of the data (first vertical or second).

Locations of 3-D sources will be offset in the direction of the intensity vector in the body. This is because the analytic signal is showing the location of maximum magnetic flux. For two-dimensional sources, the maximum flux lies over the edges of the body, except where overlapping anomalies shift this location.

Analytic contains no information about the dip of the source, or the magnetization direction of the source.

## Analytic Signal References

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