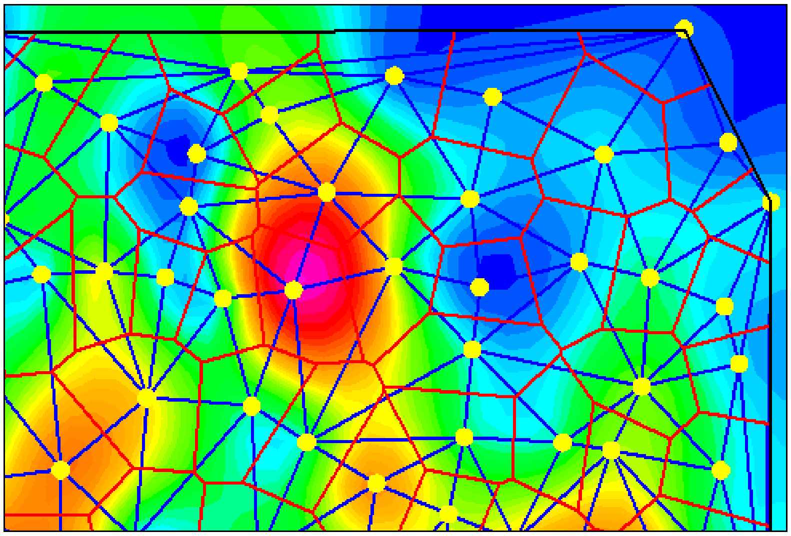
Tinning

Triangular Irregular Network Gridding for Oasis montaj

TECHNICAL NOTE



GEOSOFT

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TIN Gridding in Oasis montaj

The ability to create a TIN (Triangular Irregular Network), and to use this TIN to grid data using the "Natural Neighbours" method has just been added to the **Oasis montaj™** environment.

The TIN is created from a set of spatial data using the public domain *Sweepline* algorithm implemented by Steven Fortune of Bell Laboratories (Fortune, S 1987). The TINDB GX applies the *Sweepline* algorithm to the X, Y (Z-optional) data values in a Geosoft database (*.gdb) to create a binary TIN (*.tin) file.

When Z values are included in the (*.tin) file, a TIN grid can be created using the TINGRID GX. The TINGRID GX applies the *Natural Neighbour* algorithm (Sambridge, Brown & McQueen 1995) to the Z values in the (*.tin) file to create a grid.

The TIN gridding method requires one data point for each (X, Y) data location in the database. Tinning provides the ability to sum or average duplicate samples – data that have multiple Z values at single point locations. (Note that, when Z values are included in the (*.tin) file, only data point locations with non-dummy Z values are included.)

Geosoft Tinning provides a number of ways of visualizing the TIN, including the ability to plot the TIN Nodes, the TIN Mesh (or Delaunay triangulation), the Convex Hull, and the Voronoi cells of your data. These are described below.

Tin Nodes

TIN Nodes are the X,Y locations of the processed data. In a Z-valued TIN, required for gridding, only those locations with valid X, Y and Z values are included in the TIN.

The TINNODES GX plots the TIN Nodes.

Voronoi Cells

Voronoi cells are polygonal zones surrounding each node. Any location inside the zone is closer to the its "own" node than to any other node.

Figure 1, shows the Voronoi diagram for a set of 16 nodes. Each region, or cell, consists of the part of the plane nearest to that node. The cells are unique and can be defined similarly in any dimension (Sambridge, Brown & McQueen 1995).

The TINVORONOI GX plots the Voronoi cells.

Delaunay triangulation (Tin mesh)

The Delaunay triangulation is the set of triangles formed out of the connections between nodes in the TIN, determined by the TIN algorithm.

The Delaunay triangulation (Tin mesh) of the 16 nodes in Figure 1, is shown in Figure 2. Simply connecting the nodes of the Voronoi cells that have common boundaries forms the Delaunay triangles. In effect, the Voronoi cells and Delaunay triangles are "different sides of the same coin", and, mathematically, either can be derived from the other.

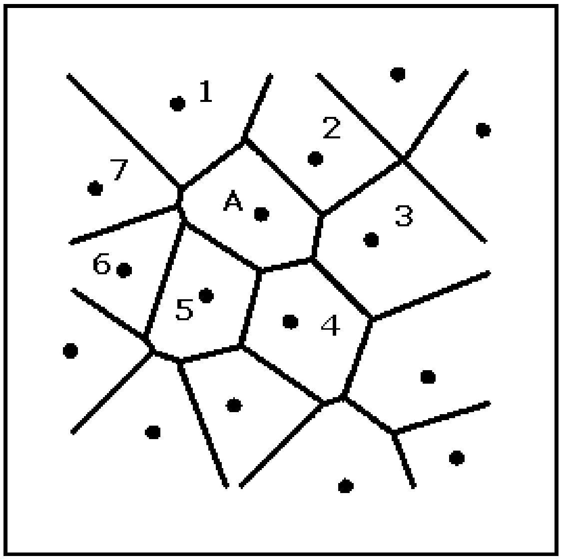


Figure 1. The Voronoi diagram for a set of 16 nodes in a plane.

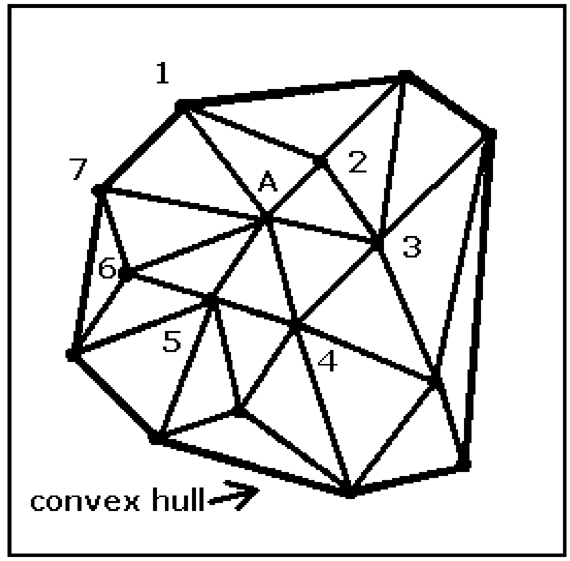


Figure 2. The corresponding Delaunay triangulation (tin mesh) of Figure 1. The thick 'perimeter' line connects the nodes in the convex hull.

Delaunay triangles are the set of least 'long and thin' triangles that can be generated among the many triangulations that are possible with irregularly distributed points. This is referred to as the *maximum – minimum angle property* and can be used as the basis for calculating the set of triangles (Fortune 1992).

Another useful property, the size of the Delaunay triangles, is strongly determined by the density of the nodal distribution. This property is derived directly from the Voronoi cells, whose area can be used as an inverse measure of the nodal density. The size of the Delaunay triangles will vary enormously when the nodal distribution is highly irregular.

The combined properties of the maximum-minimum internal angle and density-dependent size make Delaunay triangles ideal for use as the basis of an interpolation procedure.

The TINMESH GX plots the Dealunay triangles.

Convex Hull

The Convex Hull is the smallest convex set of nodes that enclose all nodes. For two-dimensional models, the line joining all nodes in the convex hull forms an outer perimeter, as shown in Figure 2.

The TINMESH GX has an option which allows you to highlight the convex hull.

Gridding with Natural Neighbours

The natural neighbours of any node are those in the neighbouring Voronoi cells, or equally, those to which the node is connected by the sides of Delaunay triangles. For example in Figure 1, node A has natural neighbours numbered 1 to 7. The importance of natural neighbours is that they represent a set of 'closest surrounding nodes' whose number and positions are well defined and vary according to the local nodal distribution.

Natural neighbours can be thought of as a unique set of nodes that defines the 'neighbourhood' of a point in the plane. If the distance between nodes is relatively large in some places, or small in others, the set of natural neighbours will reflect these features, but still represent the best set of nearby surrounding nodes. They are therefore ideal candidates for the basis of a local interpolation scheme; that is we have, from Sambridge et al (1995):

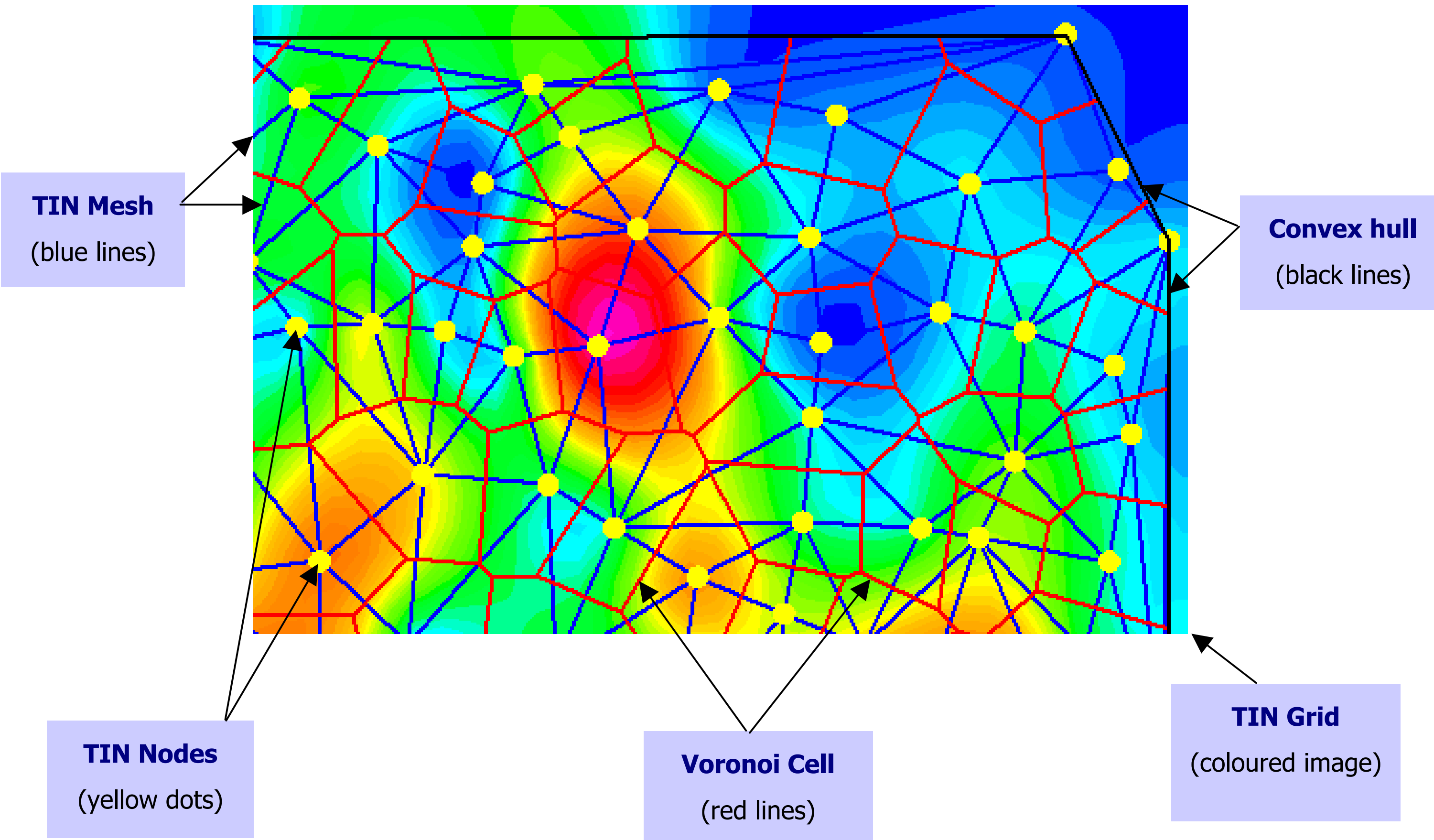
$$f(x,y) = \sum_{i=1}^{n} \phi_i(x,y) f_i, \qquad \textbf{Equation 1}$$

Where:	$f\ (x,y)$	is the interpolated function value
	$f_i\ (i=1\ldots.,n)$	is the data values at the n natural-neighbour nodes to the point (x,y)
	$\phi_i\ (I=1,\ldots..,n)$	is the weight associated with each node.

The method in which the weights, ϕ_i, are determined controls the smoothness properties of the interpolation. However, since the summation in (Equation 1), is only over the natural-neighbour nodes then, regardless of how the ϕ_i, are determined, the interpolation is guaranteed to be local. Furthermore, the size and shape of the region that can influence any point will adapt naturally to the local variation in node density. The natural neighbour grid has the desirable property that locations which fall exactly on a TIN nodes have exactly the value of that node, and the interpolation is smooth in the first derivative.

The TINGRID GX takes as input a TIN (*.tin) file to create a TIN grid.

Example of Tinning and Tin Gridding in Oasis montaj



References

Fortune, S. 1987. "A sweepline algorithm for Voronoi disgrams", Algorithmica, 2, Pages 153 –174.

Sambridge, M; Braun, J; McQueen, H. 1995. "Geophysical parametrization and interpolation of irregular data using natural neighbours". Geophysical Journal International. 122; 3, Pages 837-857.