GEOSOFT

Geosoft Technical Note

Physical Significance and Application of Fractional Vertical Derivatives

Introduction

The use of vertical derivatives has always been a standard method of enhancing high frequency features in potential field data. Second order vertical derivatives were initially computed using convolution filters and Laplace's equation. Specifically by the following relationships:

$$\nabla^2 f = 0 \text{ so,}$$

$$\frac{\partial^2 f}{\partial z^2} = -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

Equation 1

Once Fourier filtering techniques became available it was possible to compute n[th] order vertical derivatives by using the following relationship:

$$F\left(\frac{\partial^n f}{\partial z^n}\right) = k^n.F\left(f\right)$$

Equation 2

Where *F* is the Fourier representation of the field and *k* is the wavenumber or frequency.

The derivation of equation 2 from the spatial representation (equation 3) may be found in Blakeley or any graduate level geophysical text.

$$\frac{\partial f}{\partial z} = \lim_{\Delta z \to 0} \frac{f(x, y, z) - f(x, y, z - \Delta z)}{\Delta z}$$

Equation 3

Where *f* is the potential field (or it's derivative) and a *z* positive down sign convention applies. Physically this implies that the data is upward continued an infinitesimal distance and this is subtracted from the original field.

The simple form of equation 2 allows the use of non-integer values for *n*, this can than provide us with fractional vertical derivatives which have an intermediate frequency content as compared to integer order vertical derivatives.

In the frequency domain, the impulse response function for filters with varying values of *n* takes the form depicted in figure 1. Hence it's clear that high frequency components of the data are amplified to varying degrees by this class of filter.

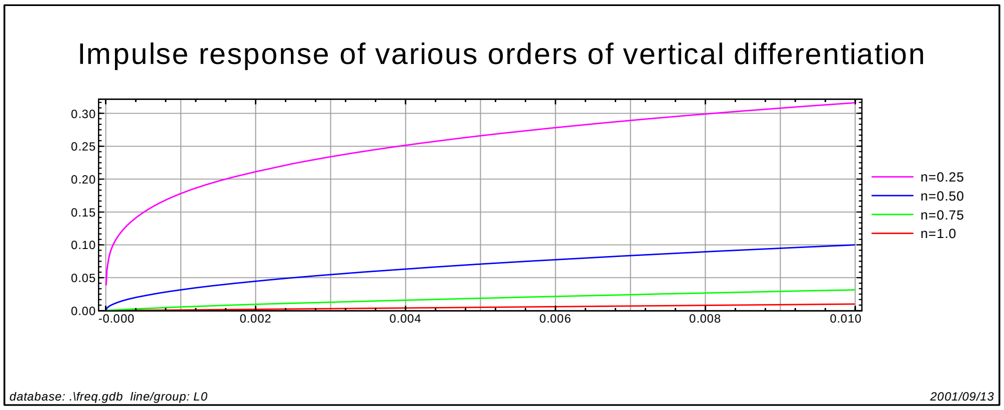

Figure 1.

A common method of regional/residual removal involves upward continuation of the data and subtracting this result from the original field. This is equivalent to applying equation 3 with a different limit on Δz and ignoring the scale factor. In this way a desired level of "smoothing" may be achieved. Figure 2 show the effect on the vertical gradient if the limit of Δz is changed.

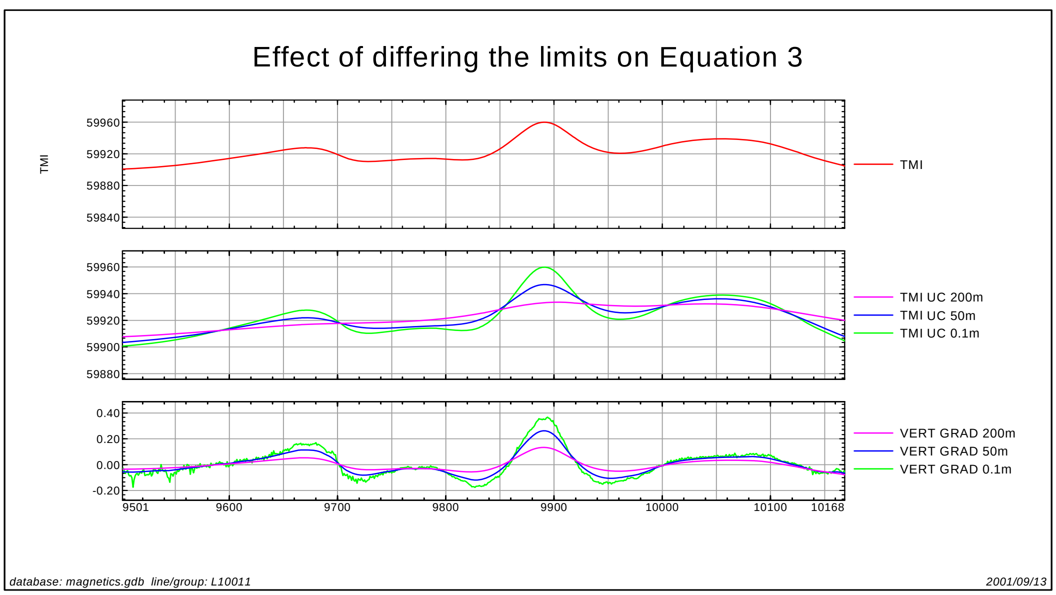

Figure 2.

From figure 2 it is clear that by increasing the limit of Δz in equation 3, the level of upward continuation is effectively increased and differing levels of regional/residual separation may be obtained.

In the frequency domain this is equivalent to using a non-integer ($0<n<1$) value of n in equation 2. If intermediate values of n are used and $1<n<2$, then the function in the previous equations refers to the first vertical gradient.

In practice fractional vertical derivatives may be used for the following:

- regional/residual separation, where changing the degree of differentiation removes less or more low frequency signal
- enhancing high frequencies in poor quality data i.e. if a first vertical derivative shows poor signal to noise ratio then a derivative of order 0.5 may enhance geology without enhancing the noise

Figures 3-6 show the effects of changing the value of n for a TMI grid.

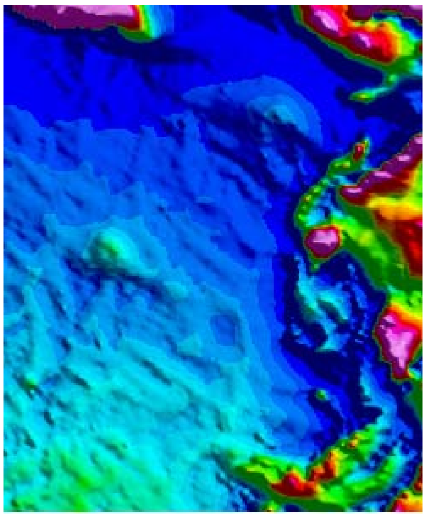

Figure 3. TMI Image

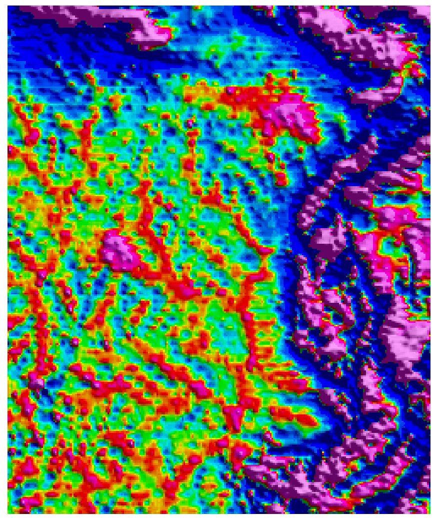

Figure 4. First vertical derivative of Figure 1. Note that, horizontal noise is evident.

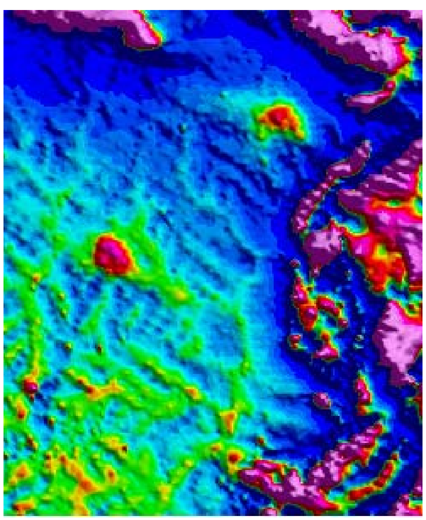

Figure 5. Vertical derivative of order 0.5. Note that, subtle high frequency features have been enhanced by a removal of regional signal.

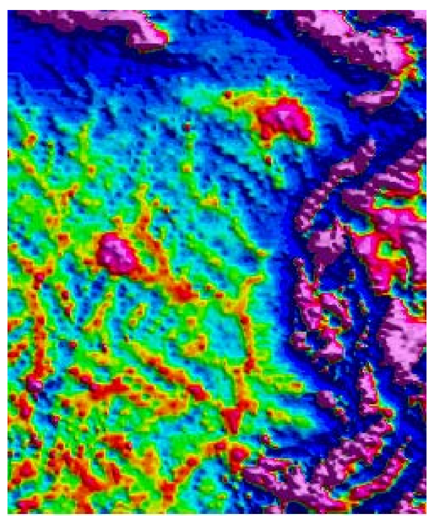

Figure 6. Vertical derivative of order 0.75. This image has the character and much of the detail of the first vertical derivative, but without the horizontal noise.

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