Quasi3D Inversion of Airborne EM Data

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SUMMARY

Full 3D inversion of AEM data is not generally available to minerals explorers because of limitations in current algorithms and computer resources. Consequently we must resort to approximations to full 3D AEM inversion to support today's exploration projects. One form of approximation is to reduce the dimensionality of the inverse problem from 3D to 1D and while layered earth inversion has proven fast and effective in practice, it has limitations in 3D environments. To address these limitations we propose a physically motivated approximate 3D AEM inversion: *Quasi3D* inversion. Full 3D EM inversion requires calculation of the 3D induced current in the earth whereas the Quasi3D approximation is based on a full 3D inversion but with a simplified, approximate, induced current flow in the earth. We demonstrate the Quasi3D approximation by comparing its response over the interface of a quarter-space model with the full AEM response, and then demonstrate Quasi3D inversion on a challenging synthetic model and on field data. From our work we conclude that the Quasi3D approximation is an effective and efficient approximation which should aid in the interpretation of AEM data for today's exploration projects.

Key words: airborne EM inversion, Quasi3D, AEM, LEI, Dighem.

INTRODUCTION

The principles of AEM forward modelling are extremely well studied for layered earth models (e.g. Kaufman and Keller, 1983). Consider a time harmonic vertical magnetic dipole source with angular frequency ω and strength *m* located at height *h* above a layered conductivity $\sigma(z)$. The time varying source magnetic field creates an azimuthal electric field in the earth given by (e.g. Ward and Hohmann, 1988, p 209)

$$E_\varphi(\boldsymbol{r}) = \frac{-i\omega\mu m}{4\pi}\int_0^\infty [e^{-u_0(z+h)} + r_{TE}e^{u_0(z-h)}]\frac{\lambda^2}{u_0}J_1(\lambda\rho)\,d\lambda \quad (1)$$

where $u_0 = \sqrt{\lambda^2 - i\omega\mu\sigma_0}$, and r_{TE} is the TE-mode reflection coefficient at the surface of the model. This field drives an azimuthal current loop in the conducting earth which in turn gives rise to the familiar H_z field measured in a horizontal coplanar coil frequency domain AEM system, for example.

It is clear from (1) that there exists an earth AEM footprint from within which the majority of the H_z response arises. The simple form of (1) gives rise to the layered earth inverse (LEI) problem for AEM. Indeed, LEI's are used extensively in the practical interpretation of AEM data. Of course, LEI's breakdown in the presence of lateral conductivity inhomogeneity and it is the purpose of this work to present a simple yet effective 3D extension of LEI inversion without the full and costly construction associated with the full 3D AEM inverse problem.

METHOD AND RESULTS

Our focus is on efficient inversion however to define our approximation to the 3D AEM inverse problem we begin by considering the forward modelling problem. Using the traditional Green's function method (Hohmann, 1988) the scattered fields can be written symbolically as,

$$\begin{bmatrix}\boldsymbol{E^s}\\ \boldsymbol{H^s}\end{bmatrix}(\boldsymbol{r}') = \int_V \begin{bmatrix}\overline{\overline{\boldsymbol{G}}}_E\\ \overline{\overline{\boldsymbol{G}}}_H\end{bmatrix}(\boldsymbol{r}|\boldsymbol{r}')\cdot \boldsymbol{J^S}(\boldsymbol{r})d^3\boldsymbol{r} \quad (2)$$

where $\boldsymbol{G}$ represents the electric and magnetic Green's functions. In conductive media the scattering current is given by

$$\boldsymbol{J^S}(\boldsymbol{r}) = \boldsymbol{E^T}(\boldsymbol{r})\,\boldsymbol{\delta}\sigma(\boldsymbol{r}) \quad (2)$$

In the full 3D EM modelling, $\boldsymbol{E^T}$ is found by a very time consuming solution of a singular Fredholm integral equation of the second kind, which we attempt to avoid via the *Quasi3D* approximation outlined below.

This problem simplifies dramatically for a layered earth where the scattering current becomes strictly azimuthal,

$$\boldsymbol{J^S}(\boldsymbol{r}) = E_\varphi(\boldsymbol{r}-\boldsymbol{r}')\,\delta\sigma(z)\hat{\boldsymbol{\varphi}} \quad (3)$$

We propose the *Quasi3D* approximation for extending to 3D conductivities by approximating the full scattering current in (3) as a radially averaged scattering current,

$$\boldsymbol{J^S}(\boldsymbol{r}) = E_\varphi(\boldsymbol{r}-\boldsymbol{r}')\hat{\boldsymbol{\varphi}}\,\overline{\delta\sigma}(\boldsymbol{r}-\boldsymbol{r}') \quad (4)$$

with

$$\overline{\delta\sigma}(\boldsymbol{r}) = \frac{\int_0^\infty E_\varphi^2(\boldsymbol{r})\int_0^{2\pi}\sigma(\boldsymbol{r})\,d\varphi\,d\rho}{2\pi\int_0^\infty E_\varphi^2(\boldsymbol{r})\;d\rho} \quad (5)$$

The physical interpretation of this approximation is that the full 3D forward modelling problem is being approximated by the forward modelling of a smoothed version of the scattering current, with the smoothing scale being dominated by the AEM footprint. More to the point, a regularized 3D inverse problem can be formulated in the traditional manner to efficiently recover a 3D conductivity model, and in the process, to some extent reverse the smoothing in (5).

To illustrate the Quasi3D approximation for forward modelling, consider the simple lateral conductivity inhomogeneity of a quarter-space model with left and right quarter spaces having conductivities 50 Ωm and 200 Ωm respectively. Figure 1 shows the results of simulating the 900Hz, 7200Hz and 56KHz vertical magnetic field response for a profile at a height of 30m for the layered earth approximation (green), the full 3D calculation (black), and the Quasi3D response (orange: diamonds for in-phase, squares for quadrature). The full 3D EM response was computed using the QMR-FFT fast IE method (Ellis 2002).

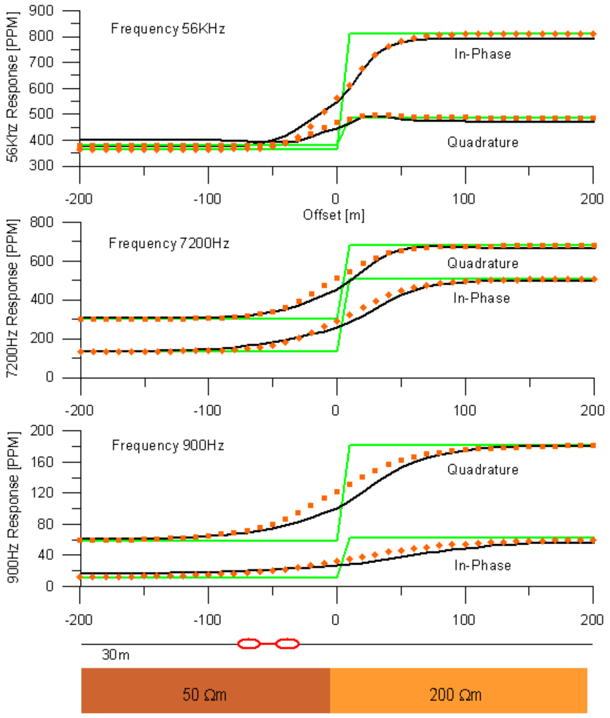

Figure 1. The Quarter space conductivity model used to compare the responses associated the layered earth approximation (green), the full 3D calculation (black), and the Quasi3D response (orange: diamonds for in-phase, squares for quadrature).

From the comparison shown in Figure 1 we can see that the Quasi3D approximation (orange) gives a reasonable approximation to the full 3D EM response (black): certainly it is very much better than strict layered earth forward modelling (green). Given this improvement over the 1D response, it is reasonable to expect that an inversion based on the Quasi3D approximation will be a significant improvement over layered earth inversion.

Incorporating the Quasi3D approximation into a 3D inversion is conceptually straightforward, albeit somewhat arduous to implement. We use a standard Tikhonov regularization (Zhdanov, 2002) of the form shown schematically in (6).

$$\text{Min } \phi(\boldsymbol{m}) = \phi_D(\boldsymbol{m}) + \lambda\, \phi_M(\boldsymbol{m})$$

$$\phi_D(\boldsymbol{m}) = \sum_j^M \left| \frac{\boldsymbol{G}_j[\,\boldsymbol{m}] - \boldsymbol{H}_j}{e_j} \right|^2 \qquad (6)$$

$$\phi_M(\boldsymbol{m}) = \sum_\gamma^3 \left| w_\gamma \partial_\gamma \boldsymbol{m} \right|^2 + |w_0 \boldsymbol{m}|^2$$

$$\lambda : \phi_D(\boldsymbol{m}) = \chi_T^2$$

where in the first line, the total objective function ϕ is the sum of a data term ϕ_D and a model term ϕ_M with a Tikohonov regularization parameter, λ. The second line defines the data objective function in terms of the non-linear forward modelling $\boldsymbol{H} = \boldsymbol{G}\,[\boldsymbol{m}]$ with $\boldsymbol{m}$ the conductivity of the earth, $\boldsymbol{G}$ the Quasi3D forward modelling operator, $\boldsymbol{H}$ the magnetic field response vector, and e_j the error associated with each data point. The third line gives the model objective function in terms of the gradient of the model $\partial_\gamma \boldsymbol{m}$ and the amplitude of the model, with weighting terms as required, w_γ, w_0. The fourth line indicates that the Tikohonv regularization parameter λ is chosen based on a satisfactory fit to the data in a chi-squared sense, χ_T^2. In addition, other constraints, such as upper and lower bounds, can be placed on $\boldsymbol{m}$ as appropriate to the specific exploration problem.

To demonstrate the Quasi3D approximation for inversion we consider an example taken from Raiche et al. (2003). The model simulates a massive sulphide ore zone in an altered ultramafic halo under a paleochannel at the boundary of two different host units under a hill as illustrated in Figure 2 (the "Sulphide" model). This rather challenging synthetic was used by Raiche et al. to demonstrate the shortcomings of LEI in somewhat realistic exploration situations and so makes a good test for the Quasi3D inversion.

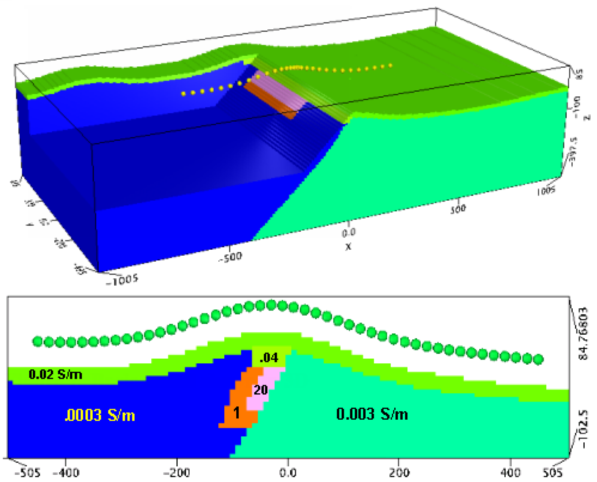

Figure 2. The "Sulphide" conductivity model used to test the Quasi3D approximation. The upper panel shows a 3D perspective, the lower panel shows a section through the centre of the model. The model consists of massive sulphide (magenta), an alteration halo(brown), a paleochannel (lime), a fault, and saprolite cover (green) under an undulating topography.

In this work we are focusing on frequency domain systems so we reduce the vertical scale of Raiche's model by a factor of 2 and simulate the frequency domain response consistent with a Dighem system at frequencies 900 Hz, 7200 Hz, and 56 KHz, with a 30 m clearance. We use a modification of the finite element program LokiAir (Raiche 2008) to compute the responses. Nine flight lines with 20m station spacing were simulated over the target, with the central line shown in Figure 2 (upper panel).

The Sulphide model Dighem data were inverted using the Quasi3D algorithm. In Figure 3 we show a section through the centre of the recovered model, analogous to that shown in Figure 2 (lower panel). We immediately see that a strong conductive target has been imaged at the correct lateral location of the ore zone, and there is a faint hint of a dip in the inversion result. As expected, the nature of regularization in (6) produces a smooth model in the absence of other constraints. Focusing methods can be used to improve the resolution however that is beyond the scope of the current work.

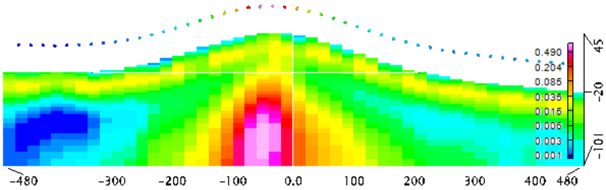

Figure 3. The result of Quasi3D inversion of the LokiAir data simulated over the Sulphide conductivity model used to compare the inversions associated with the layered earth approximation, the full 3D response, and the Quasi3D approximation.

As a final example we demonstrate Quasi3D inversion of Dighem field data over the Cerro de Maimón deposit, a 4 million tonne copper gold deposit grading 2.53% copper, 0.96 g/t gold and 34.8 g/t silver, in the Dominican Republic. It was initially owned by Falconbridge Dominicana and was later acquired by the GlobeStar Mining Corporation (GMC) in April 2002, with Perilya Limited purchasing 50% in 2008. The deposit is a sheet-like structure, the top is 30 m below the surface and the sheet dips at about 40 degrees to the south west as shown in the geological cross section in Figure 4 (Roos et al. 2008). The thickest part of the deposit is about 75 m deep.

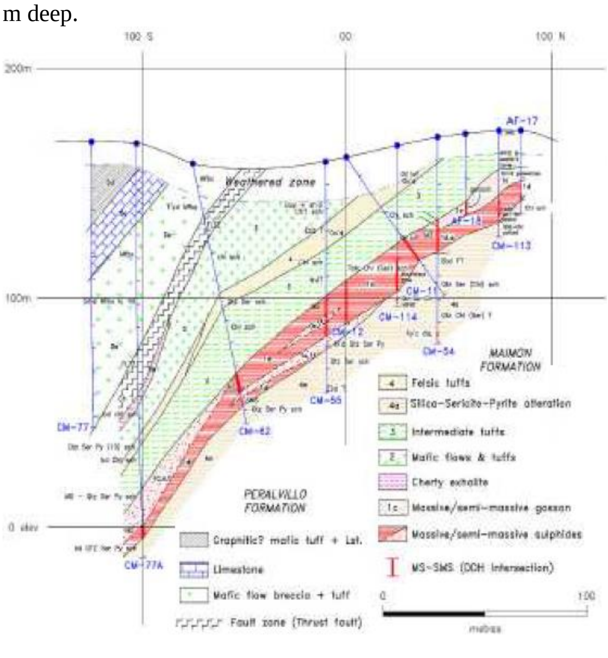

Figure 4. Cross section of the Cerro de Maimón Deposit. Sulphides are massive (red) to semi-massive (hatched). Grid lines are 100m. (Roos et al. 2008)

The HeliGeotem response over this deposit was studied by Smith and Hodges (2008). The Dighem data are shown in Figure 5 for the 900 Hz, 7200 Hz, and 56 KHz frequencies, in-phase and quadrature channels. Most of the signal in the 7200 Hz and 56 KHz is due to conductive weathering with the massive sulphide conductor appearing predominantly in the in-phase 900 Hz channels as would be expected for a very conductive target.

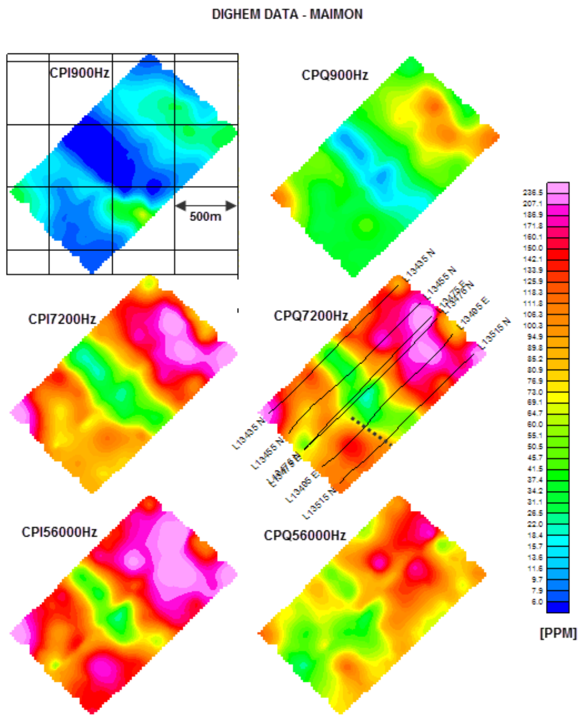

Figure 5. The Dighem data acquired over the Maimón deposit, courtesy of CGG and GlobeStar, shown for the 3 frequencies (900Hz, 7200Hz, 56000Hz), in-phase and quadrature components. The flight lines are superimposed on the CPQ7200Hz response with the surface expression of the deposit (dashed).

Inverting the Maimón data with the Quasi3D algorithm produces the 3D conductivity model shown in Figure 6 with 0.012 S/m threshold. The conducting ore zone is imaged in the correct location however there is very little indication of the dip of the target in this display. The conducting weathered cover is also evident.

In Figure 7 we show a Quasi3D model section directly under Line 13515 together with the observed data and predicted response of the recovered model. The dominant target response in the 900 Hz in-phase channel, mentioned above, is clear in the profile plot. Note that the observed and predicted data fit well for all channels. Examining the model section we see that the inversion has recovered the ore zone, with a slight hint of a dipping target. It is worth mentioning here that even full 3D AEM regularized inversion of this data will result in some smoothing of the recovered target (i.e. loss of dip) as a consequence of non-uniqueness, so at this stage we cannot attribute lack of model resolution strictly to the Quasi3D approximation per se. Unfortunately, comparison of the

Quasi3D approximation with full 3D inversion is beyond the scope of this work.

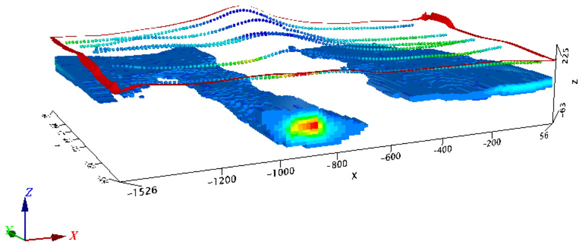

Figure 6. The voxel model recovered by Quasi3D inversion, showing the Maimón orebody (red) and the conductive weathered cover (blue). The voxel threshold is 0.012 S/m.

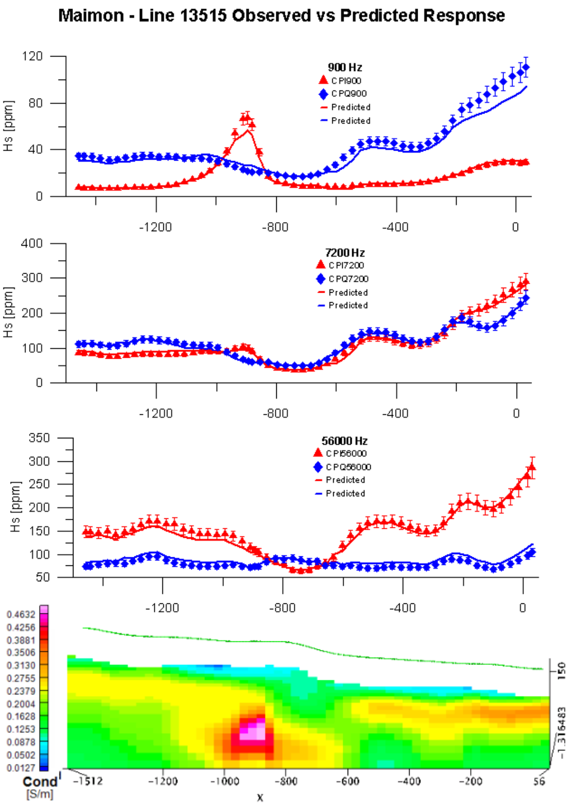

Figure 7. The observed data (symbols) and predicted data (solid) for Line 13515 are shown in the 3 upper panels for the 900 Hz, 7200 Hz, and the 56 KHz Dighem data at Cerro de Maimón. The bottom panel shows the corresponding slice through Quasi3D inversion model.

CONCLUSIONS

Full 3D inversion of AEM data is currently impractical for most exploration projects leading to the need for approximate methods to assist in data interpretation. To this end we presented the Quasi3D approximation as a natural 3D extension of LEI inversion, based on a simplification of the scattered current density. We demonstrated that the Quasi3D approximation is viable by comparing the forward model response from a full 3D AEM simulation with the Quasi3D response over the contact of a quarter-space model. We formulated the 3D AEM inverse problem in terms of the Quasi3D approximation and demonstrated that encouraging results were obtained when inverting synthetic data over a complicated model specifically designed to challenge LEI inversions. Finally we applied the Quasi3D inversion to Dighem data collected over the Cerro de Maimón deposit in the Dominican Republic. Again, encouraging results were obtained with the Quasi3D inversion. These successful results suggest that the Quasi3D approximation is a viable approximation for 3D inversion of AEM data and warrants further investigation and development.

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